

ON COMPLEX SYMMETRIC TOEPLITZ OPERATORS ON THE GENERALIZED DERIVATIVE HARDY SPACE

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ABSTRACT. Let $H(\mathbb{D})$ be the space of all functions analytic on the open unit disk \mathbb{D} . The generalized derivative Hardy space $S_{\alpha,\beta}^2(\mathbb{D})$ consists of all functions whose derivatives are in the Hardy and Bergman spaces as follows; for $\alpha, \beta \in \mathbb{N}$,

$$S_{\alpha,\beta}^2(\mathbb{D}) = \left\{ f \in H(\mathbb{D}) : \|f\|_{S_{\alpha,\beta}^2}^2 = \|f\|_{H^2}^2 + \frac{\alpha + \beta}{\alpha\beta} \|f'\|_{A^2}^2 + \frac{1}{\alpha\beta} \|f'\|_{H^2}^2 < \infty \right\}.$$

In this paper, we study necessary and sufficient conditions for Toeplitz operators to be complex symmetric on the generalized derivative Hardy space $S_{\alpha,\beta}^2(\mathbb{D})$ with respect to some conjugations. Moreover, we study complex symmetric Toeplitz operators with non-analytic symbols.

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