On Putnam Inequalities

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Abstract

Let $T = H + iK$ be hyponormal on a complex Hilbert space. Then C.R. Putnam ([9]) proved the following inequalities:

$$\|T^*T - TT^*\| \ \leq \ \frac{1}{\pi}\, m(\sigma(T)) \ \text{ and } \ \|T^*T - TT^*\| \ \leq \ \frac{2}{\pi}\, \|K\| \cdot m(\sigma(H)).$$

Since now there are various extensions, I'll explain them.

References

[1]. S.K. Berberian, *Approximate proper vectors*, Proc. Amer. Math. Soc., **13** (1962), 111-114.

[2]. M. Chō, R. Curto, T. Huruya and W. Żelazko, *Cartesian form of Putnam's inequality for doubly commuting hyponormal n-tuples*, Indiana Univ. Math. J. **49** (2000), 1437-1448.

[3]. M. Chō, B.P. Duggal and W.Y. Lee, *Putnam's inequality og doubly commuting n-tuples for* log-*hyponormal operators*, Proc. Roy. Irish Acad. 100(2000), 163-169.

[4]. M. Chō and T. Huruya, *Putnam's inequality for p-hyponormal n-tuples*, Glasgow Math. J. **41** (1999), 13-17.

[5]. M. Chō and M. Itoh, *Putnam's inequality for p-hyponormal operators*, Proc. Amer. Math. Soc., **123**(1995), 2435-2440.

[6]. M. Chō and T. Nakazi, *Spectral area estimates for norms of commutators*, J. Korean Math. Soc. **44**(2007), 855-861.

[7]. M. Chō and K. Tanahashi, *New spectral mapping theorem of the Taylor spectrum*, Science Math. Japon **84**, No.2 (2021), 145-154.

[8]. T. Nakazi, *Complete spectral area estimates and self-commutators*, Michigan Math. J., **35**(1988), 435-441.

[9]. C.R. Putnam, Commutation properties of Hilbert space operators, Springer-Verlag, 1967.

[10]. K. Tanahashi, *Putnam's inequality for* log-*hyponormal operators*, Integr. Equat. Oper. Theory, **48**(2004), 103-114.

[11]. A. Uchiyama, *Berger-Shaw's theorem for p-hyponormal operators*, Integr. Equat. Oper. Theory, **33**(1999), 221-230.

[12]. A. Uchiyama, *Inequalities of Putnam and Berger-Shaw for p-quasihyponormal operators*, Integr. Equat. Oper. Theory, **34**(1999), 91-106.

[13]. J.L. Taylor, *The analytic functional calculus for several commuting operators*, Acta Math., **125**(1970), 1-38.

[14] J.L. Taylor, *A joint spectrum for several commuting operators*, J. Funct. Anal. **6**(1970), 172-191.

[15]. F.-H. Vasilescu, *On pairs of commuting operators*, Studia Math. **62**(1978), 203-207.

[16]. D. Xia, *On the semi-hyponormal n-tuple of semi-hyponormal operators*, Integr. Equat. Oper. Theory **6**(1983), 879-898.

[17]. D. Xia, Spectral Theory of Hyponormal operators, Birkhäuser 1983.