Cyclic multiplicity of a direct sum of forward and backward shifts

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Let H_E^2 be the vector-valued (E-valued) Hardy space and S_E be the shift (multiplication by z) operator on H_E^2. We study the cyclic multiplicity of $S_E \oplus S_F^*$ (denoted by $\mu(S_E \oplus S_F^*)$). When $\dim E = \dim F = 1$, this is an old problem proposed by Herroro in 1978 and late answered by Herraro and Wogen. The answer is also given by Problem 163 in Halmos's "A Hilbert Space Problem Book" where an elegant and short proof is attributed to Nikolskii, Peller and Vasunin. We extend the idea of Nikolskii, Peller and Vasunin to prove that $\mu(S_E \oplus S_F^*) = \dim E + 1$. Our proof uses a construction of outer functions which is the main obstacle to extend this result to vector-valued weighted Bergman spaces. Using recent invariant subspace theorem of $S_E \oplus S_F^*$, we also study the question of when $Span\{h_i : 1 \leq i \leq n\}$ is not a cyclic subspace of $S_E \oplus S_F^*$.

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