Conditionally positive definite unilateral weighted shifts

Il Bong Jung

Let $\gamma = \{\gamma_n\}_{n=0}^{\infty}$ be a sequence of real numbers. We say that γ is *positive definite* if $\sum_{i,j=0}^{k} \gamma_{i+j}\lambda_i\bar{\lambda}_j \geq 0$ for all finite sequences $\lambda_0, \ldots, \lambda_k \in \mathbb{C}$. If this inequality holds for all finite sequences $\lambda_0, \ldots, \lambda_k \in \mathbb{C}$ such that $\sum_{j=0}^{k} \lambda_j = 0$, then we call the sequence γ *conditionally positive definite* (CPD for brevity). Let $\mathcal{H}$ be an infinite dimensional complex Hilbert space and let $B(\mathcal{H})$ be the algebra of all bounded operators on $\mathcal{H}$. An operator $T \in B(\mathcal{H})$ is said to be *conditionally positive definite* (CPD) if each sequence of the form $\{\|T^n h\|^2\}_{n=0}^{\infty}$ is CPD for all $h \in \mathcal{H}$. In this paper we characterize the conditional positive definiteness of unilateral weighted shifts W_λ with weight sequence $\lambda = \{\lambda_n\}_{n=0}^{\infty}$ in terms of formal moment sequences, and give the description of the representing triplet of W_λ which is the main object canonically associated with such operators. Recall that several operator theorists have studied the backward extension and flatness problems in the case of other operator classes such as subnormal, n-hyponormal and weakly n-hyponormal weighted shifts, etc., since 1990' for more than 30 years. We solve the backward extension problem and flatness problem for CPD unilateral weighted shifts. In particular, the flatness problem in this context is discussed with an emphases on unexpected differences from the analogous problem for subnormal unilateral weighted shifts.

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Department of Mathematics, Kyungpook National University, Daegu 41566, Korea

E-mail: `ibjung@knu.ac.kr`