The mixed identity for normed modules

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The "mixed identity" for tensor products of (unital) modules over (unital) rings says that there is isomorphism

$$Hom_B(N \otimes_A M, P) \cong Hom_A(M, Hom_B(N, P))$$

whenever M is a left (A, B) bimodule, N is a (right A, left B) bimodule and P a left B module. For normed modules over a normed algebra holds for bounded linear mappings,

$$BL_B(N \otimes_A M, P) \cong BL_A(M, BL_B(N, P))) \ ,$$

provided norm of the tensor product is that derived from the "greatest cross-norm". Taking P and B to be the scalar field, this specialises to

$$M^* \cong BL_A(M, A^*) \ .$$

Spin-off says that the real and the complex duals of a complex normed space coincide, half of the von Neumann double commutant theorem extends from Hilbert ot reflexive Banach spaces, and the Arens multiplication on the second dual is explained.

This trawl into my ancient history is my contribution to a joint project with Dragan Djordjevic.

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