Limit of the iteration of induced Aluthge transformations of centered operators

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Let H be a complex Hilbert space, and $\mathcal{B}(H)$ be the C^*-algebra of all bounded linear operators on H. For the polar decomposition $T = U|T|$ of an operator $T \in \mathcal{B}(H)$, the Aluthge transformation $\Delta(T) = |T|^{\frac{1}{2}}U|T|^{\frac{1}{2}}$ is well known, and it has a lot of nice properties. On the other hand, S.H. Lee, W.Y. Lee and J. Yoon defined the mean transform $\hat{T} = \frac{|T|U+U|T|}{2}$. Very recently, we defined an extension of these maps which is called the induced Aluthge transformation. It depends on operator means, for example, $\Delta(T)$ and $\hat{T}$ are the induced Aluthge transformations with respect to the geometric and arithmetic means, respectively. In this talk, we shall introduce convergence of iteration of the induced Aluthge transformations. Precisely, (i) if T is an invertible semi-hyponormal operator (i.e., $|T^*| \leq |T|$), then the iteration of the induced Aluthge transformations with respect to an arbitrary operator mean converges to a normal operator, and the limit points are the same, (ii) if T is an invertible centered matrix (i.e., $\{|T|, U^n|T|U^{*n}, U^{*m}|T|U^m : n, m = 1, 2, ...\}$ is a commuting set), then the iteration of the induced Aluthge transformations with respect to an arbitrary operator mean converges to a normal operator, and the limit points are given, and (iii) we discuss the limit point of the induced Aluthge transformation with respect to the power mean in the injective II_1-factor $\mathcal{M}$ and determine the form of its limit of some centered operators in $\mathcal{M}$.

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