Mathematical and Numerical Optimization, 3341.454
E. Ryu
Fall 2020

Additional Problems for Homework 2

Problem 1: *A general vector composition rule.* Suppose

$$f(x) = h(g_1(x), g_2(x), \ldots, g_k(x))$$

where $h : \mathbb{R}^k \to \mathbb{R}$ is convex, and $g_i : \mathbb{R}^n \to \mathbb{R}$. Suppose that for each i, one of the following holds:

- h is nondecreasing in the ith argument, and g_i is convex
- h is nonincreasing in the ith argument, and g_i is concave
- g_i is affine.

Show that f is convex. (This composition rule subsumes all the ones given in the book, and is the one used in software systems such as CVX.) You can assume that $\mathbf{dom}\, h = \mathbb{R}^k$; the result also holds in the general case when the monotonicity conditions listed above are imposed on $\tilde{h}$, the extended-valued extension of h.