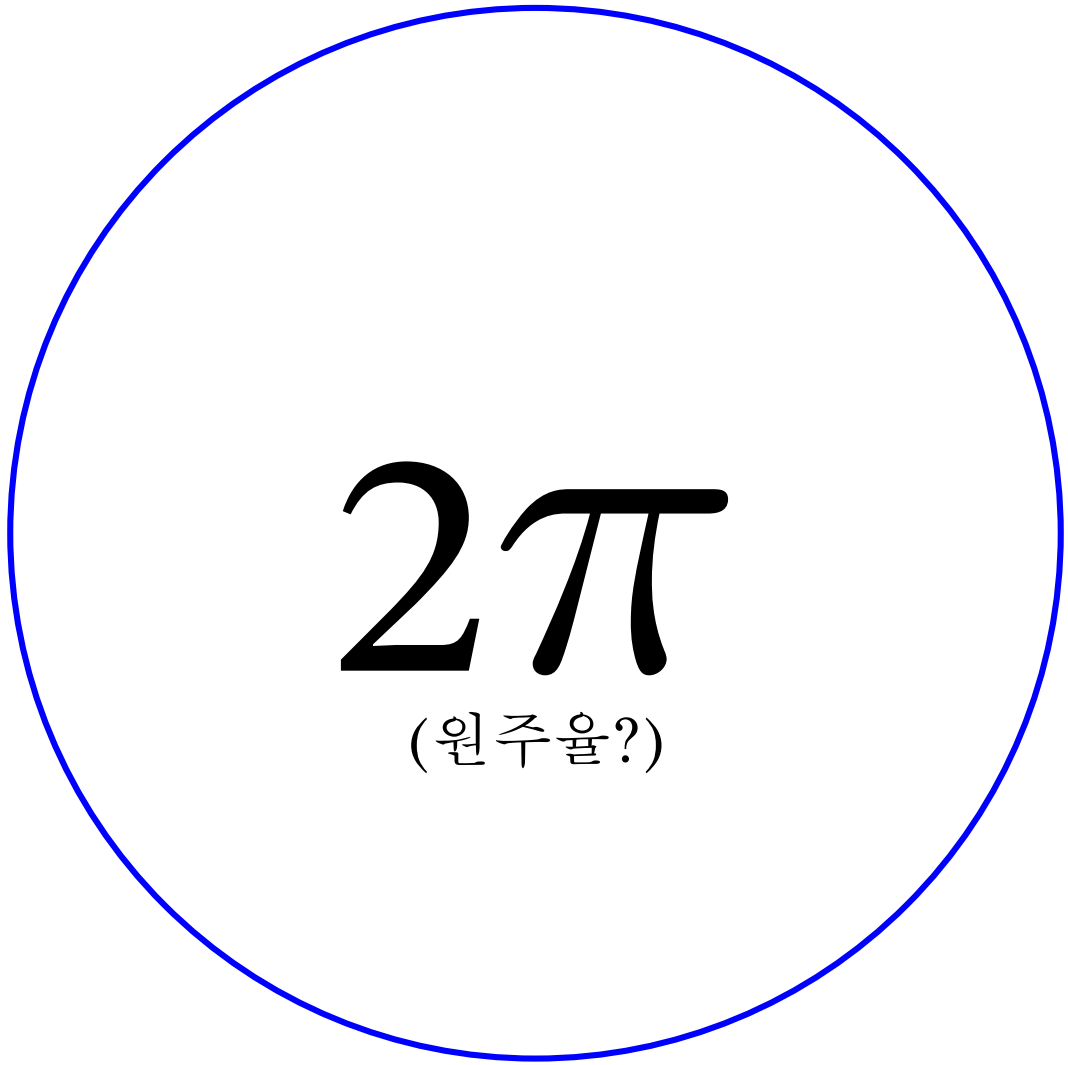
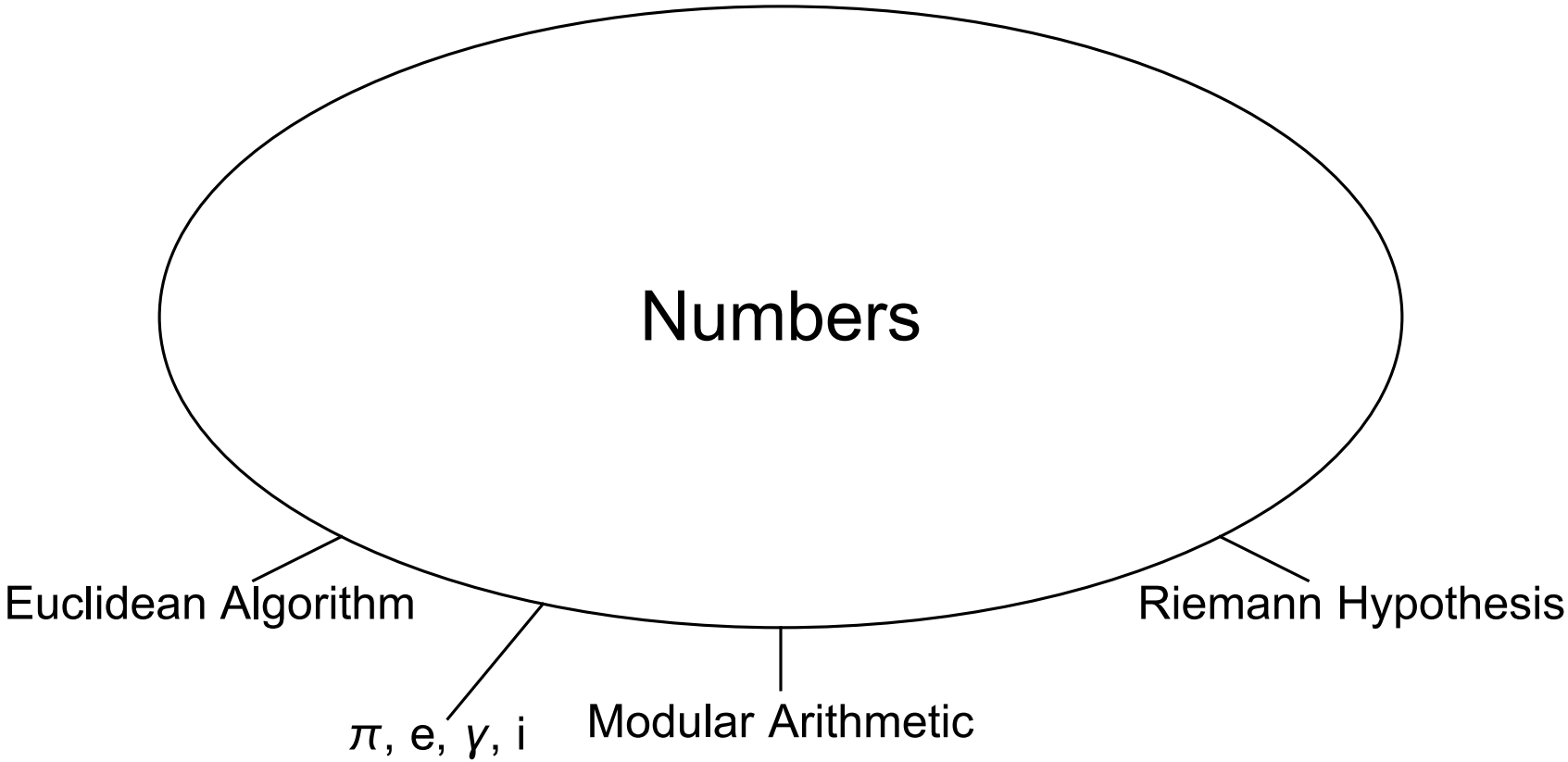
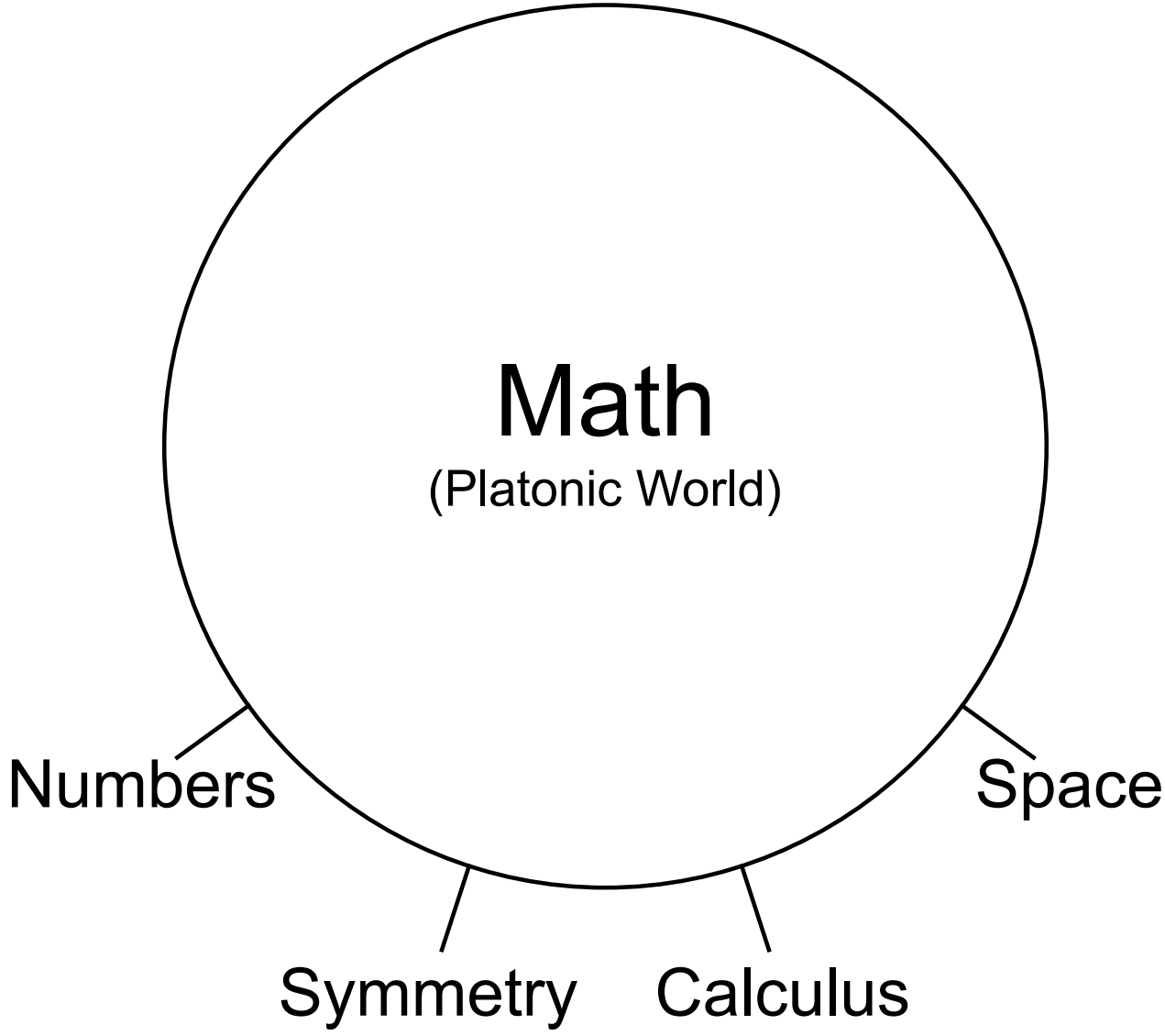


Kosmos Club, 2026. 3. 14. 김홍종

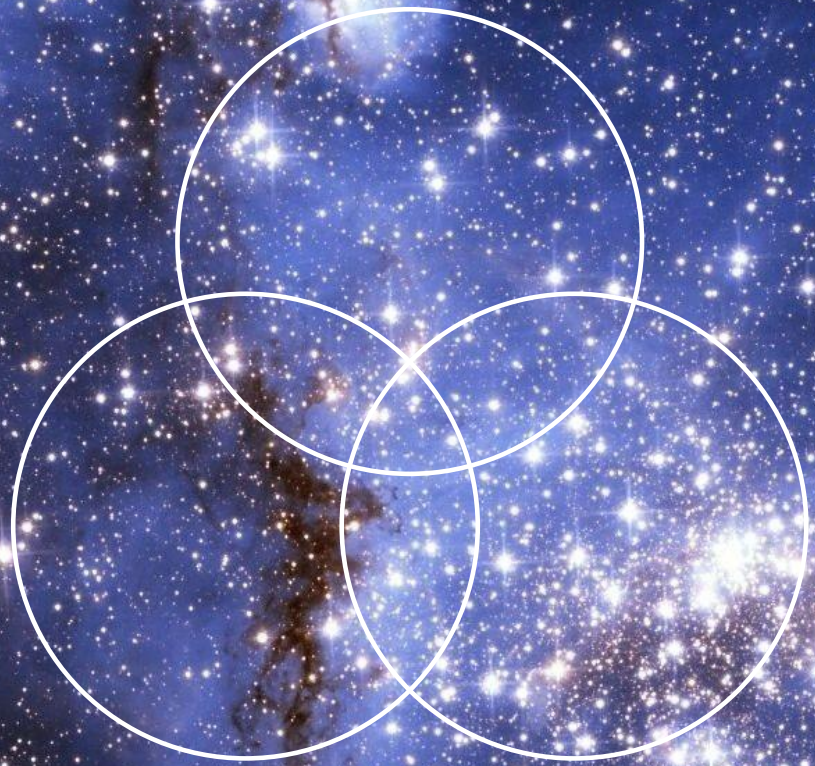

Numbers
Euclidean Algorithm
π, e, γ, i
Modular Arithmetic
Riemann Hypothesis



논문·곰국

K O S M O S

(Project $\mathcal{M}$)



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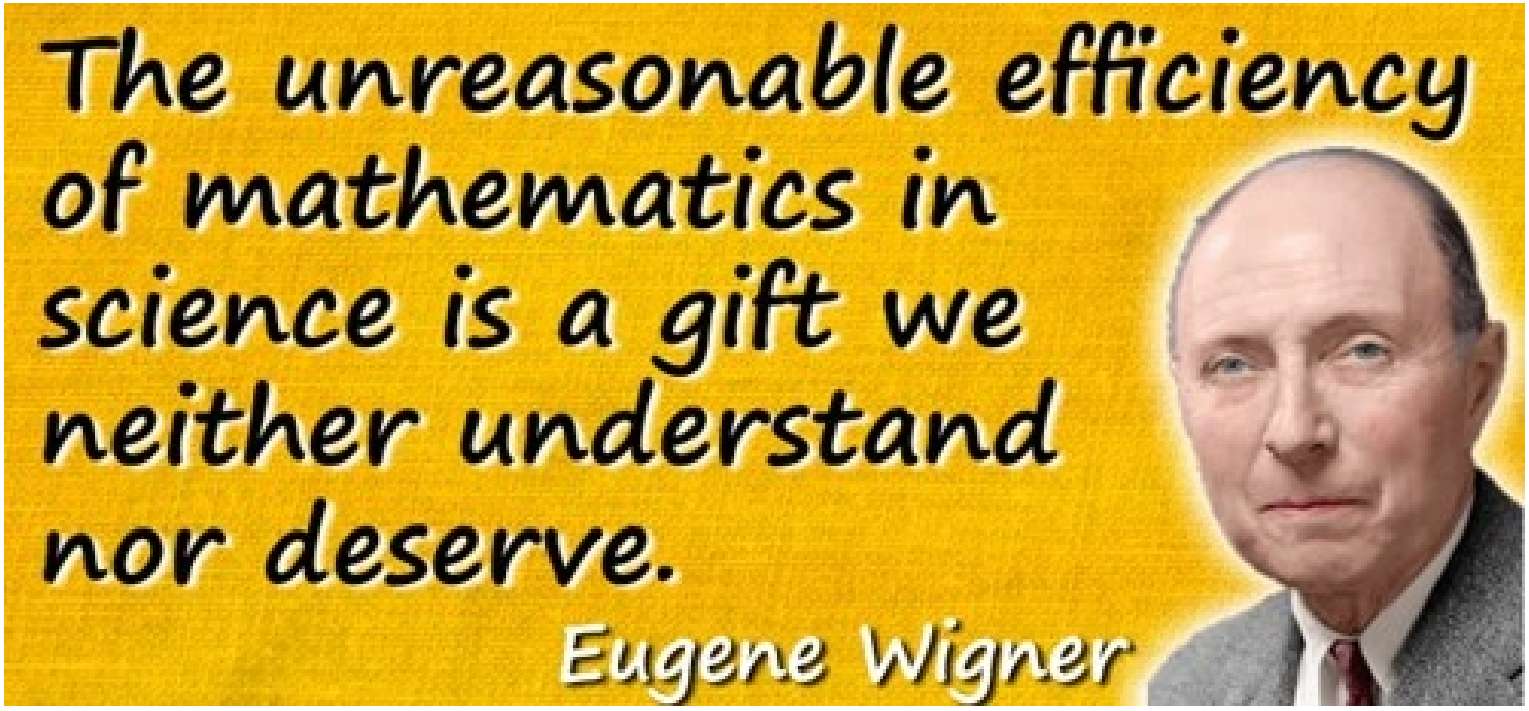
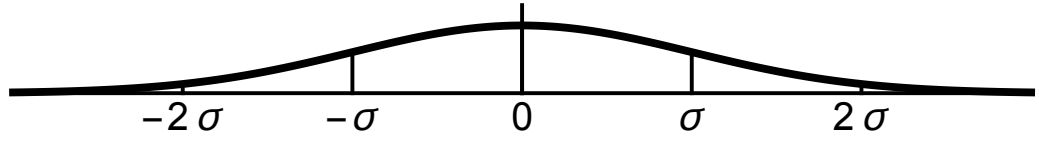
Eugene Wigner, 1902–1995



$$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$



Surely the population has nothing to do with the circumference of the circle.

1960, *The Unreasonable Effectiveness of Mathematics in the Natural Sciences*

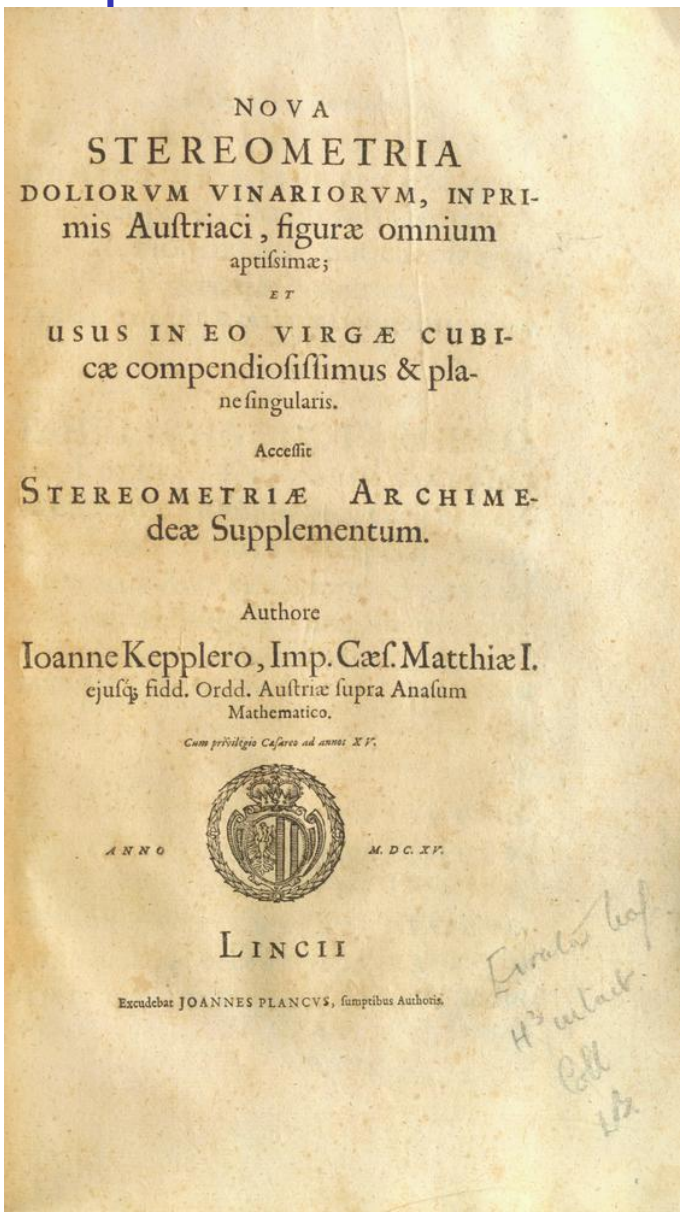
M. Plack, P. Dirac, Heisenberg

$$h = 6.62607015 \times 10^{-34}\,\mathsf{m}^2\,\mathsf{kg}\,/\,\mathsf{s}$$

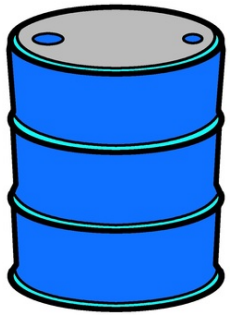
$$\hbar = \frac{h}{2\pi}$$

$$\Delta x \cdot \Delta p \geq \tfrac{1}{2}\hbar \qquad \text{(uncertainty principle)}$$

J. Kepler



NOVA
STEREOMETRIA
DOLIORVM VINARIORVM, INPRIMIS Auſtriaci, figuræ omnium aptiſsimæ;
ET
USUS IN EO VIRGÆ CUBICÆ compendioſiſſimus & plane ſingularis.
Acceſſit
STEREOMETRIÆ ARCHIMEDEÆ Supplementum.
Authore
Ioanne Kepplero, Imp. Cæſ. Matthiæ I. ejuſq; fidd. Ordd. Auſtriæ ſupra Anaſum Mathematico.
Cum privilegio Cæſareo ad annos XV.
ANNO M. DC. XV.
LINCII
Excudebat JOANNES PLANCVS, ſumptibus Authoris.



Nova Stereometria Doliorum Vinariorum
(New Solid Geometry of Wine Barrels)

by Kepler, 1615

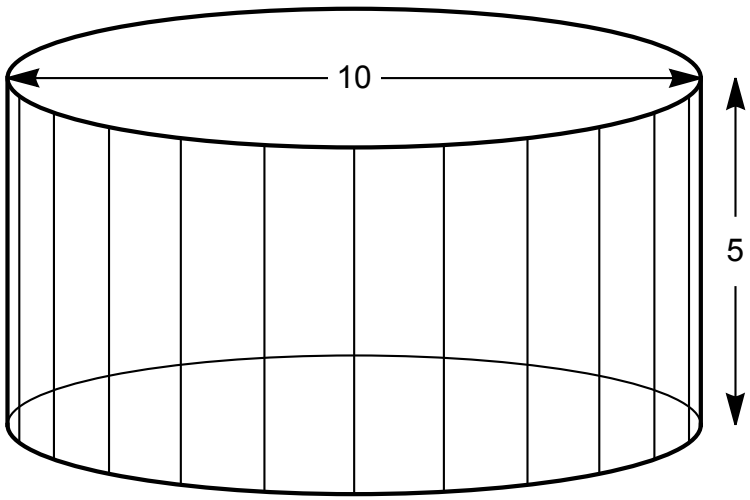
$$\text{(Volume of Revolution)} = \pi \int r^2 \, \mathrm{ds}$$

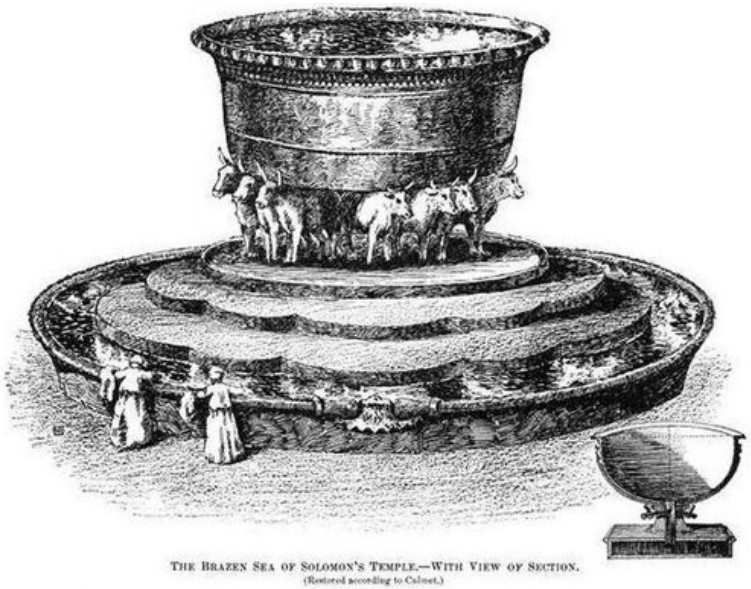
Roberto Cardil, *Kepler: The Volume of Wine Barrel*, 2011, MAA

Odometer & Speedometer

구약성서 열왕기상 (1 King 7:23), 연대기 성경 (2 Chronicles 4:2)

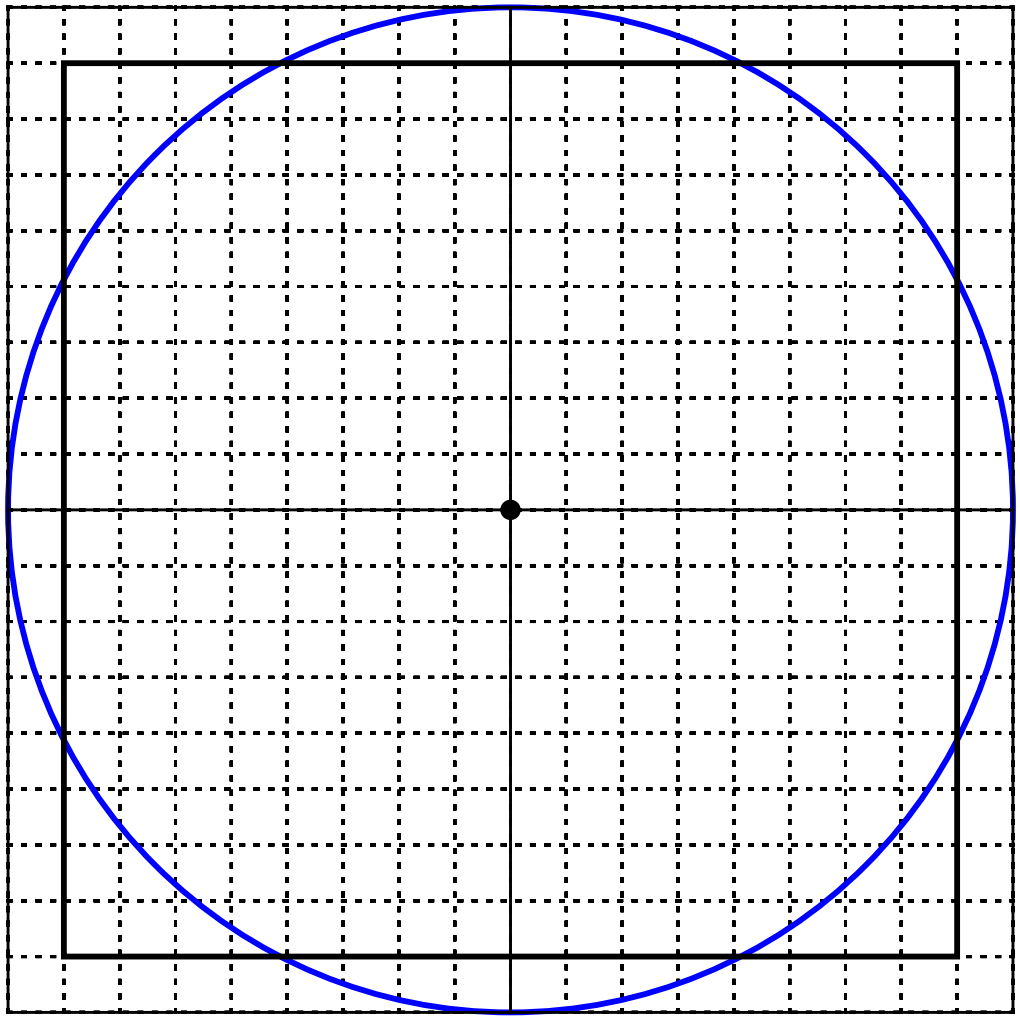
He (Solomon) made the Sea of cast metal, circular in shape, measuring ten cubits from rim to rim and five cubits high. It took a line of thirty cubits to measure around it.





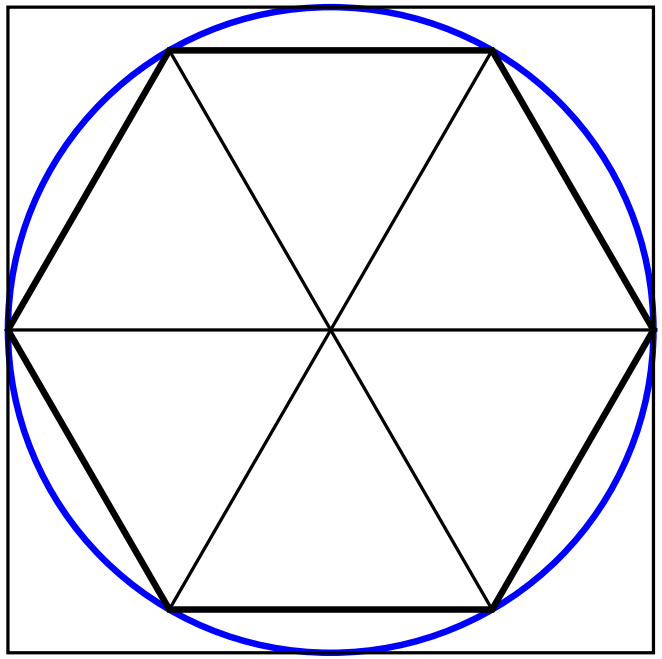
https://en.wikipedia.org/wiki/Molten_Sea

Ahmes (or Rhind) Papyrus, 1550 BCE



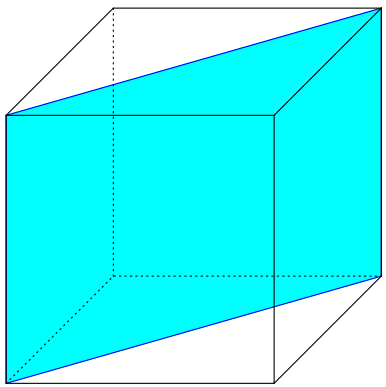
$$4 : \pi \approx 9^2 : 8^2 \quad \Rightarrow \quad \pi \approx \frac{2^8}{3^4} \approx 3.16$$

Archimedes



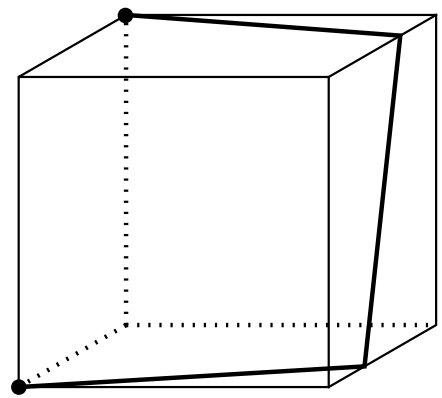
$$6 < 2\pi < 8, \qquad (3 < \pi < 4)$$

$$3\frac{10}{71} < \pi < 3\frac{1}{7} = 3.\dot{1}42857\dot{}$$



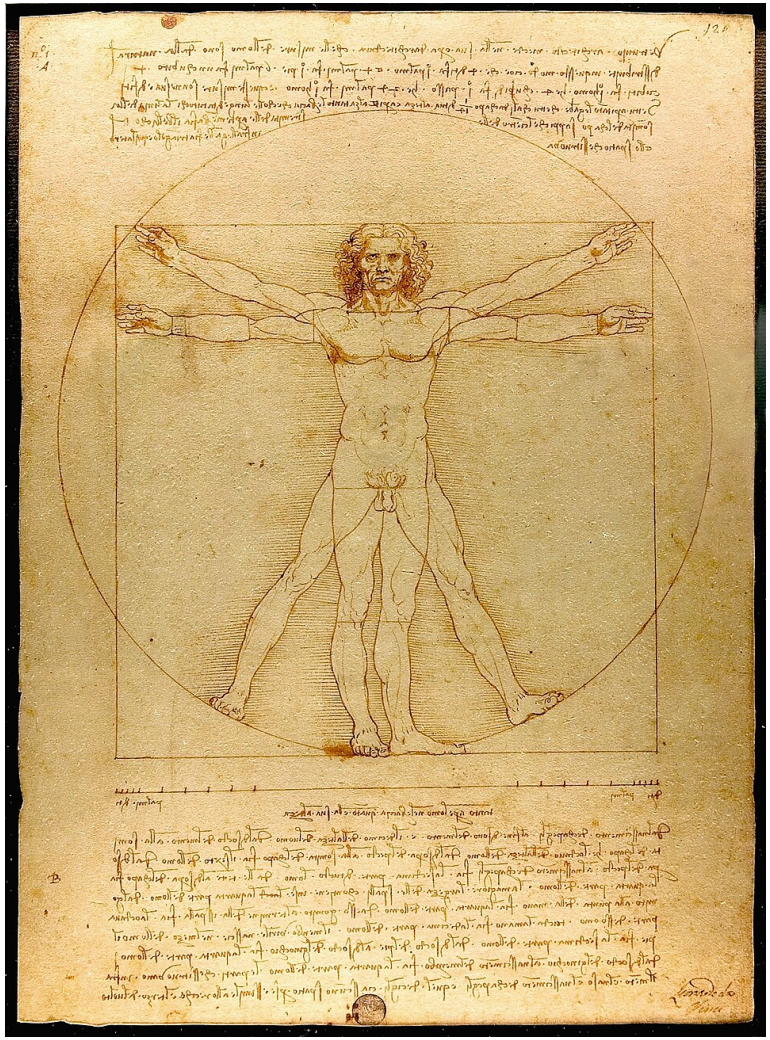
Copy-Paper (A4) in Cube

$$\sqrt{2} + \sqrt{3} \approx 3.14626$$

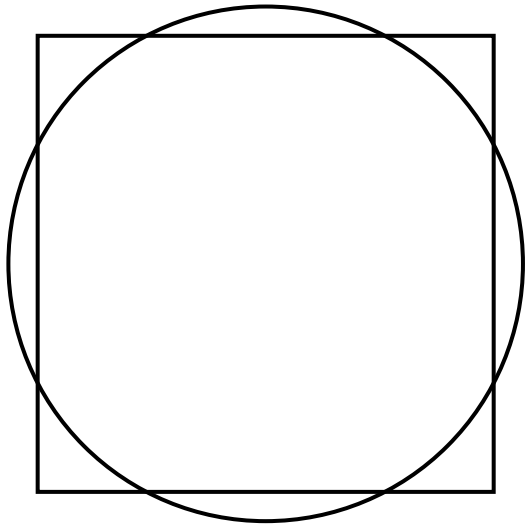


$\sqrt{10} \approx 3.16228$

Squaring the Circle Problem



Vitruvian Man by Leonardo da Vinci
(全裸男圖)



- 1882, Lindemann:
  π *is transcendental.*
- 1925, Alfred Tarski's
  `Circle Squaring Problem`:
  Is it possible to take a disc in the plane, cut it into finitely many pieces, and reassemble the pieces so as to get a square of equal area?
- 1990, Miklós Laczkovich, Hungary:
  Yes!

Exercise

Show that the disk

$$D := \{z \in \mathbb{C} : |z| \leq 1\}$$

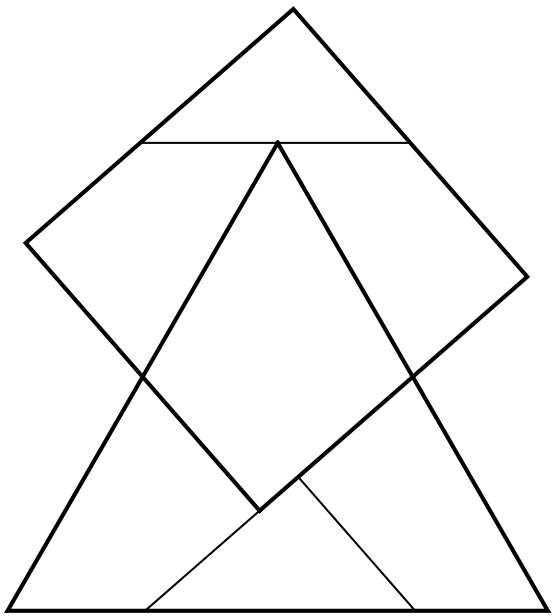
and

$$D_o := \{z \in \mathbb{C} : 0 < |z| < 1\}$$

are equidecomposable.

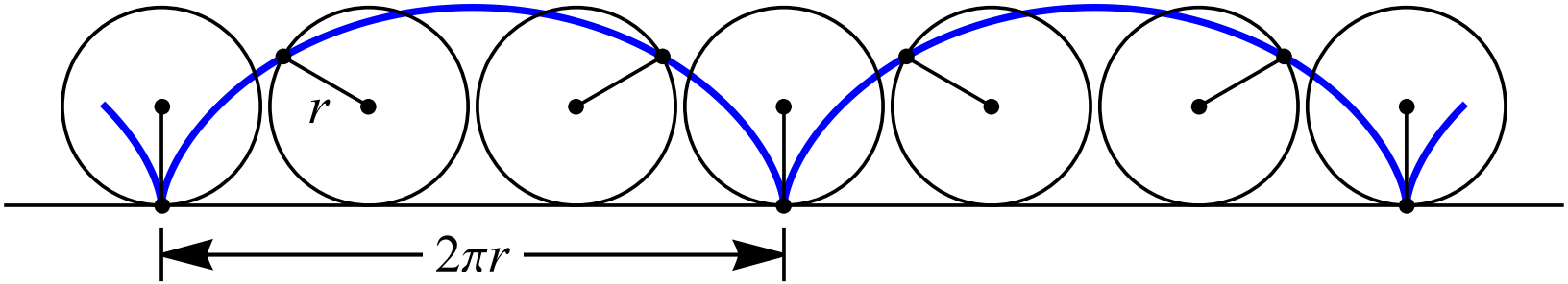
Equidecomposable

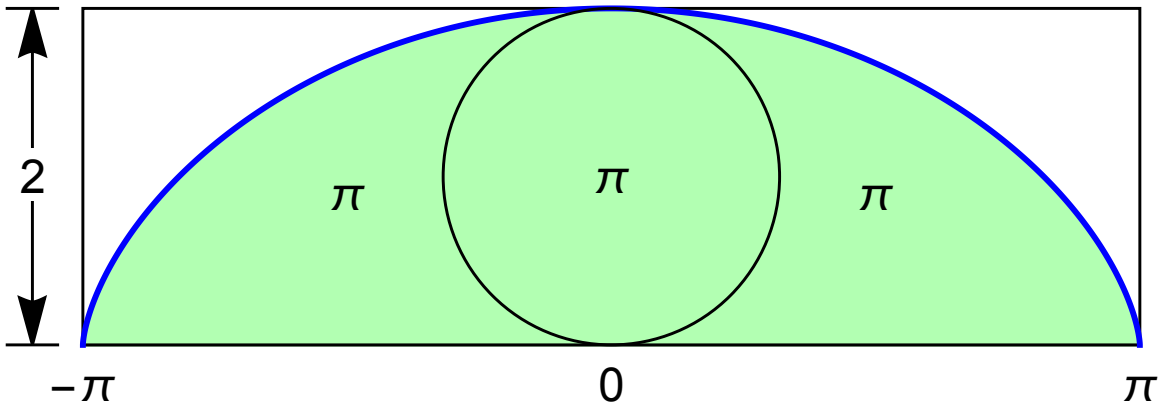
Theorem (Wallace 1807, Bolyai 1833, Gerwien 1835): *Any two polygons are equi-decomposable by polygons iff they have the same area.*



Theorem (Dehn, 1900): *A tetrahedron and a cube are NOT equi-decomposable by polyhedra.*

Cycloid, Helen of Geometers

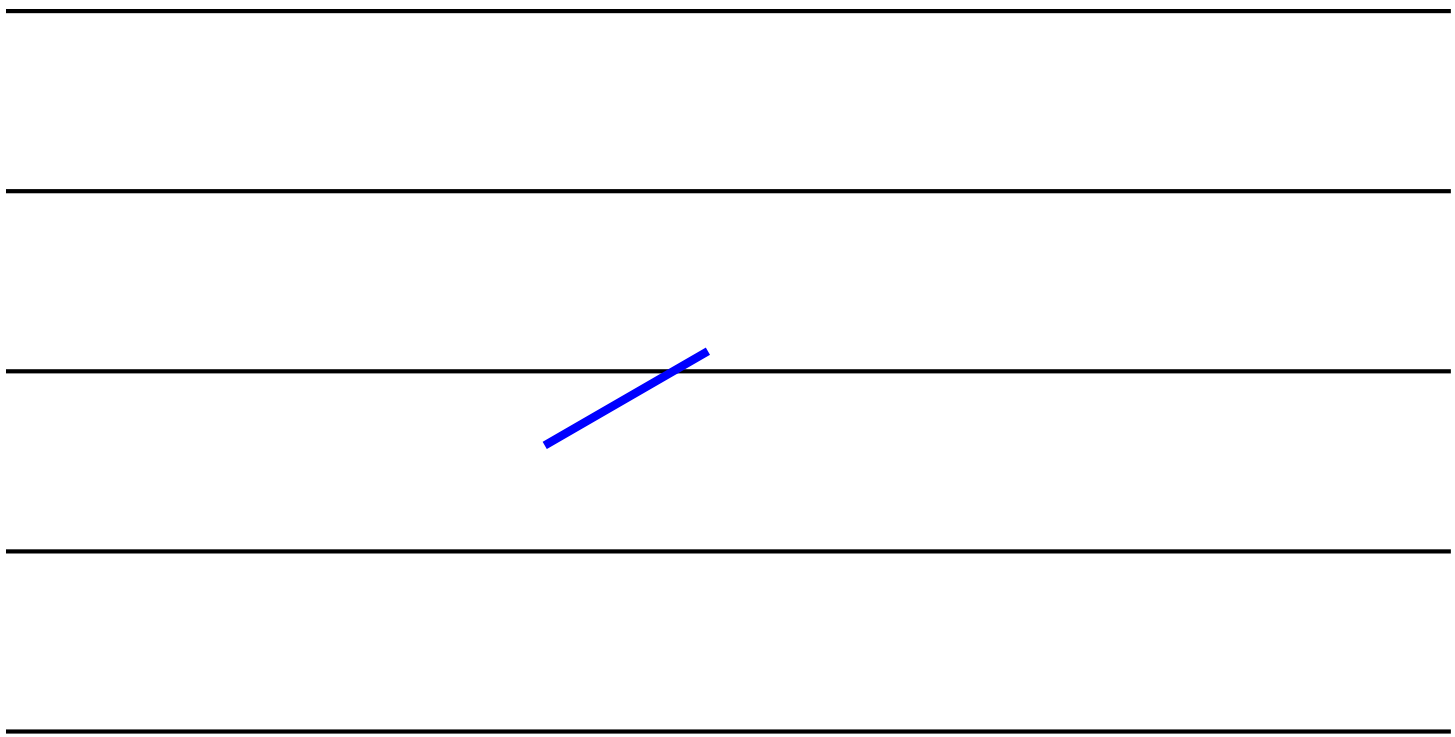




See [Bellos] or [Posamentier and Geretschläger] to find out why the cycloid is called the *Hellen of Geometers*. Cycloid was studied by Dürer, Galileo, Roberval, Torricelli, Pascal, C. Huygens (1629–1695), Newton, Leibniz, Johann Bernoulli etc., and is also called the *apple of discord* [Eves].

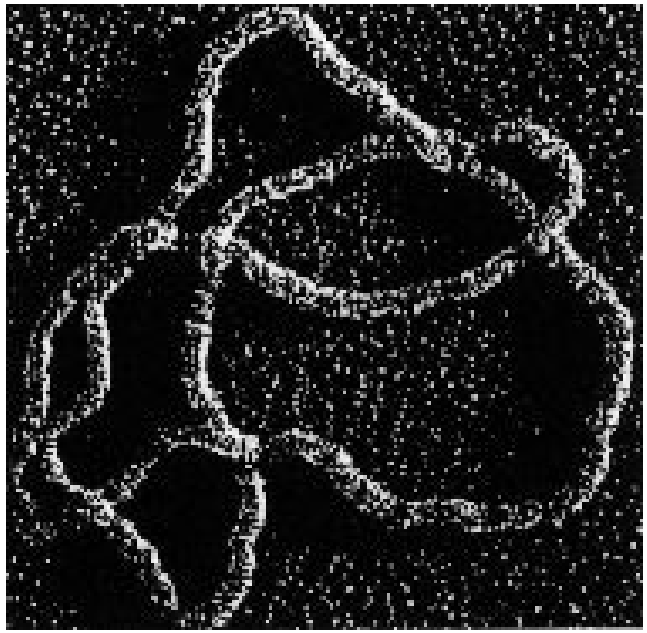
The cycloid is a *tautochrone* and also a *brachistochrone*. Cycloid is a kind of *roulette*, i.e., a curve rolling, without slipping, along another fixed curve.

Buffon's Needle (1707–1788)



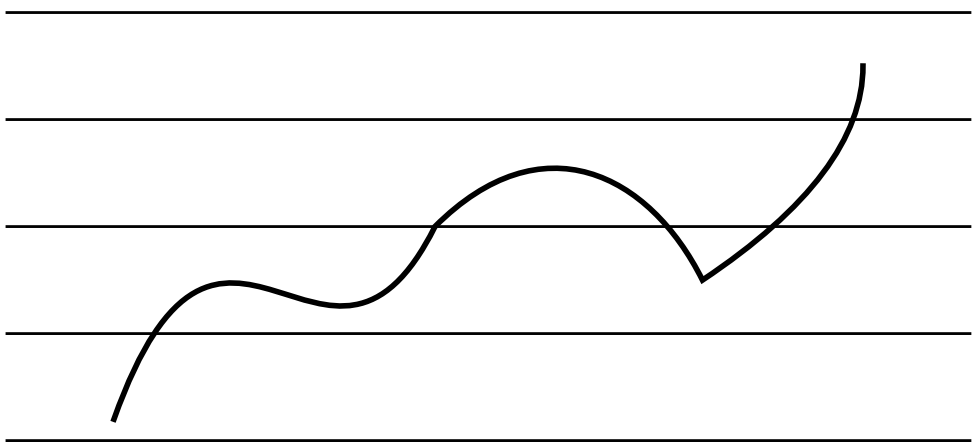
$$\text{(Probability of hitting a line)} = \frac{2}{\pi} \approx 64\,\%$$

DNA 4-plat



Noodle

Evenly spaced parallel lines are drawn on the plane.
Throw a planar string Γ on the plane.
Let $E(\#\Gamma)$ be the expected number of crossings with the lines.



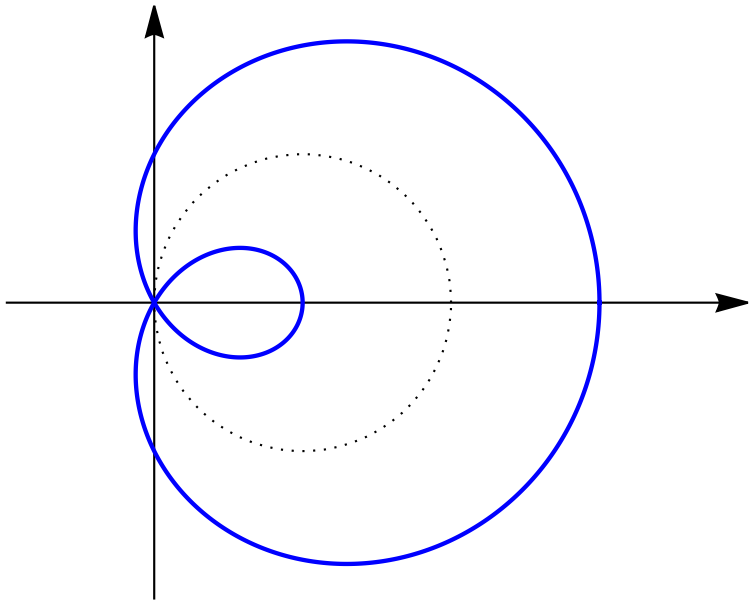
$\#\Gamma = 5$

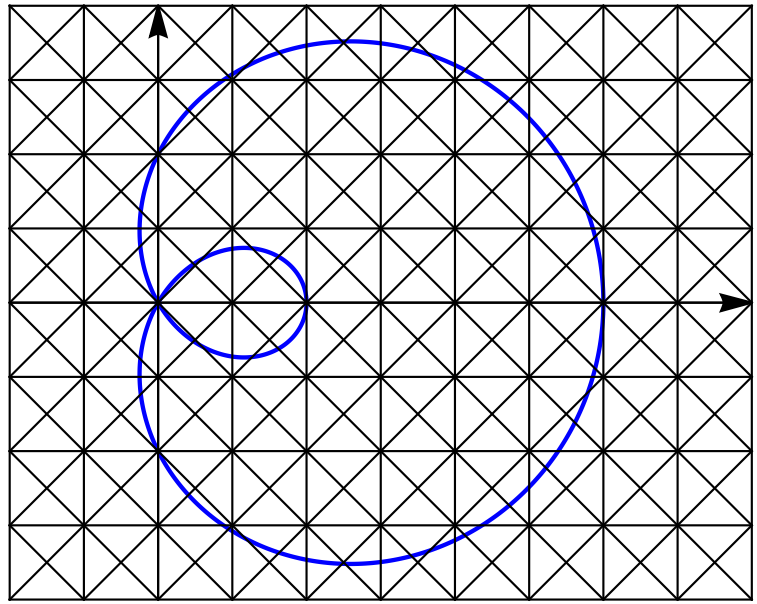
Then the length of Γ with respect to the distance between adjacent lines is

$$L(\Gamma) = \frac{\pi}{2} \times E(\#\Gamma)$$

Corollary: If Γ is an *oval*, then $L(\Gamma) = \pi \cdot E(\text{width } \Gamma)$.

Limaçon

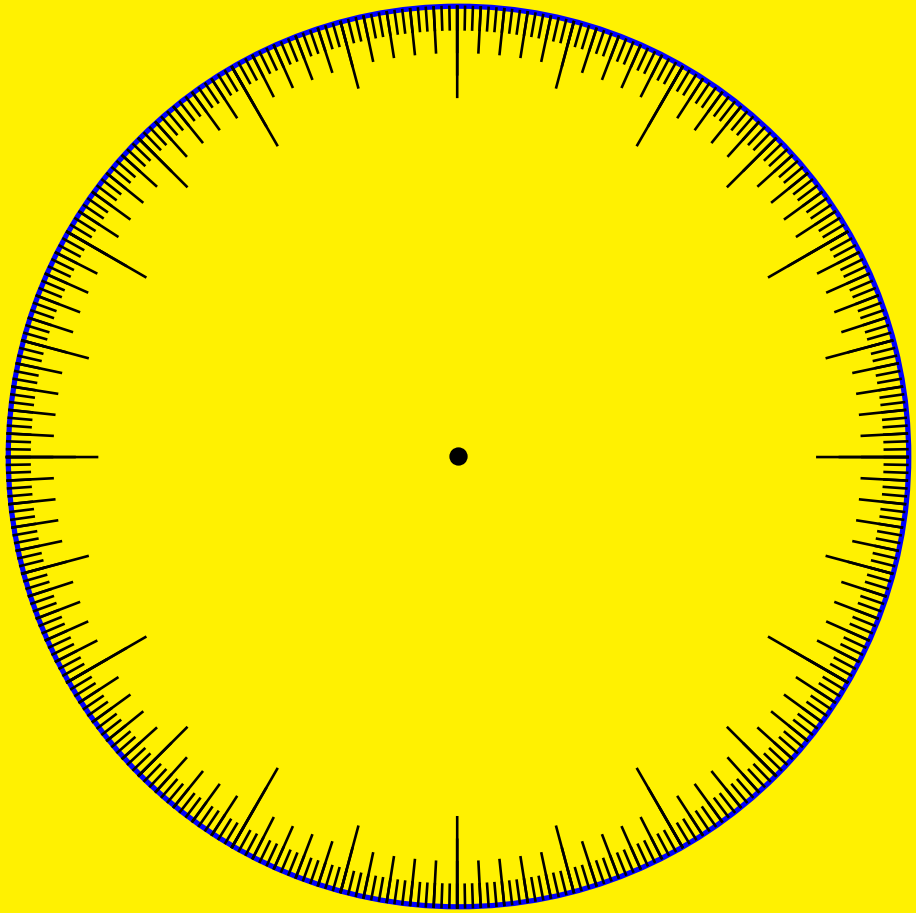




$$r = 2\cos\theta + 1, \qquad L = \int_0^{2\pi} \sqrt{5 + 4\cos\theta}\, d\theta$$

Limaçon is often called the limaçon of Étienne Pascal (1588–1651), the father of the famous Blaise Pascal (1623–1662) [Boyer]. But this curve was describe by the German artist Albrecht Dürer (1471–1528) in his four Books on Measurement *Underweysung der Messung mit dem Zirckel und Richtscheyt* (Instructions for Measuring with Compass and Ruler), 1525. Limaçon is a trisectrix and a kind of *conchoid* of Nicomedes (c. 200 BCE) with respect to a circle and a point on the circle.

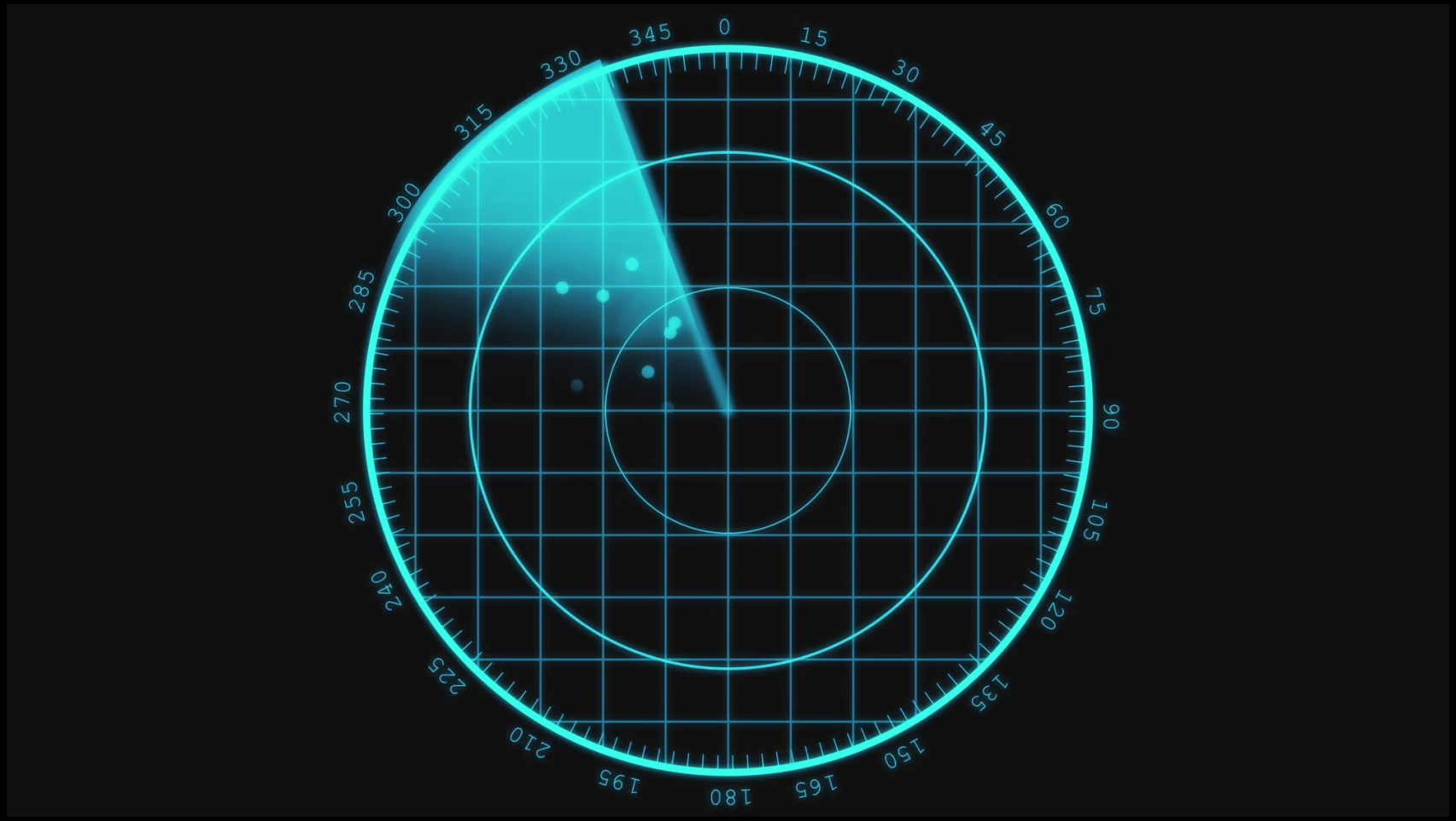
Angles and Time



하늘, 하루, 한해

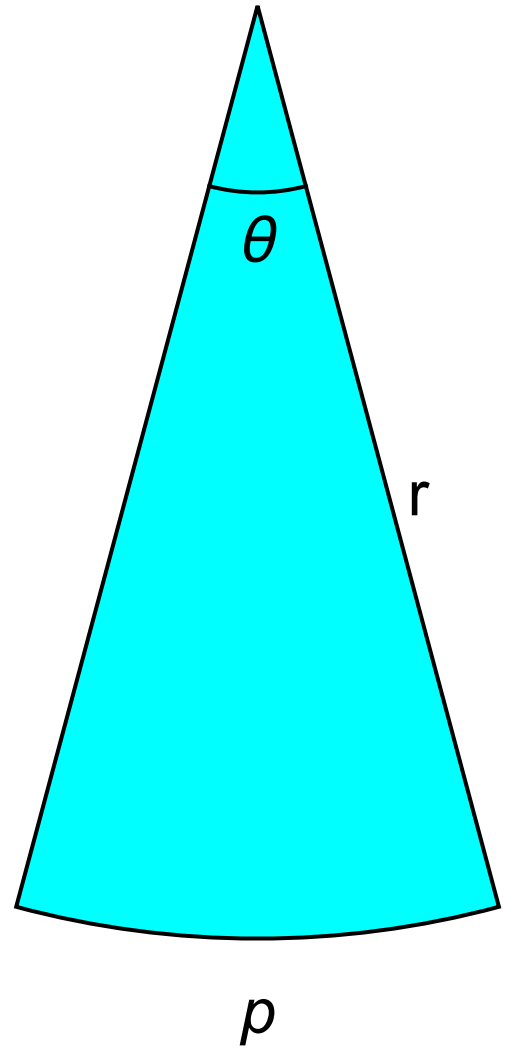
Construct 3°, and discuss cyclotomy!

Radar

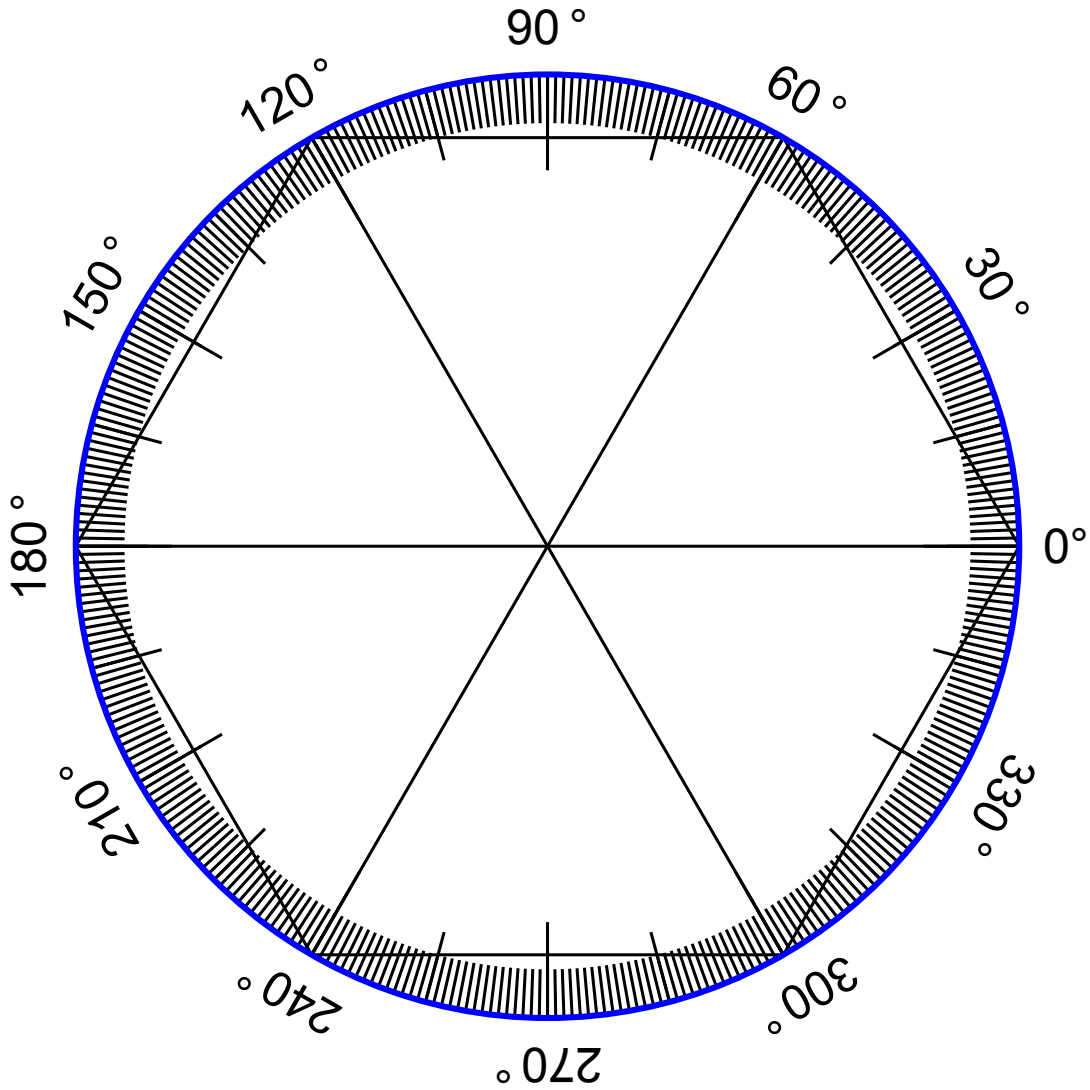

345
0
15
30
45
60
75
90
105
120
135
150
165
180
195
210
225
240
255
270
285
300
315
330

What is the measure of an angle?

$$\theta = \frac{p}{r}$$



Sexagesimal System

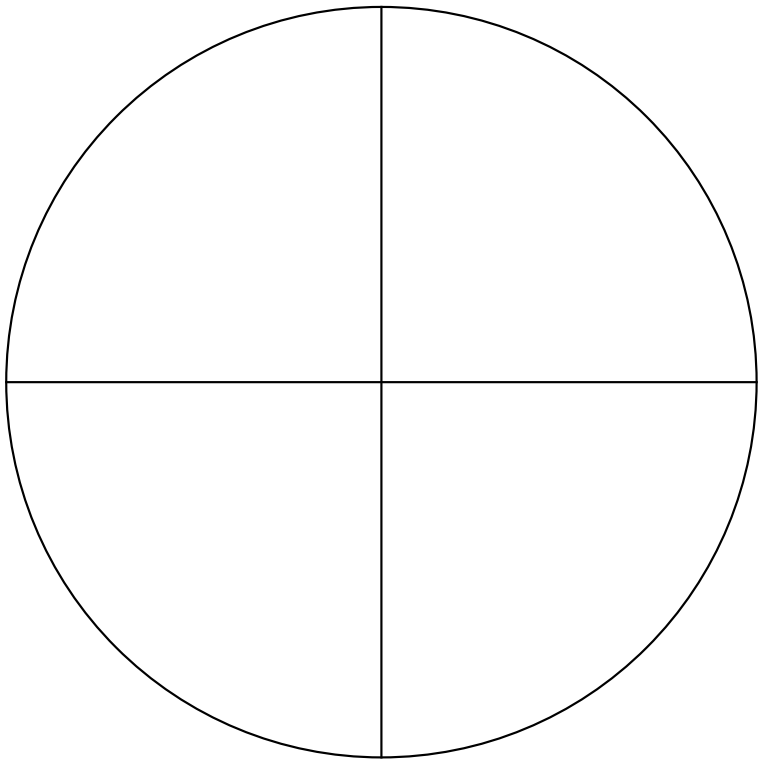


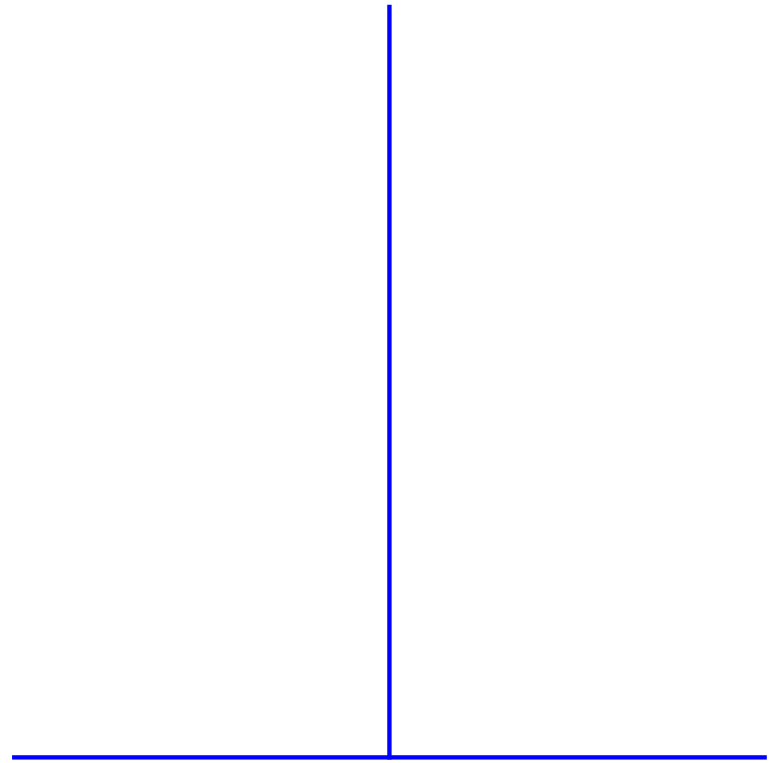
(Sun $\approx$ Moon $\approx 0.5^\circ$)

Obelisk

용인 세중박물관

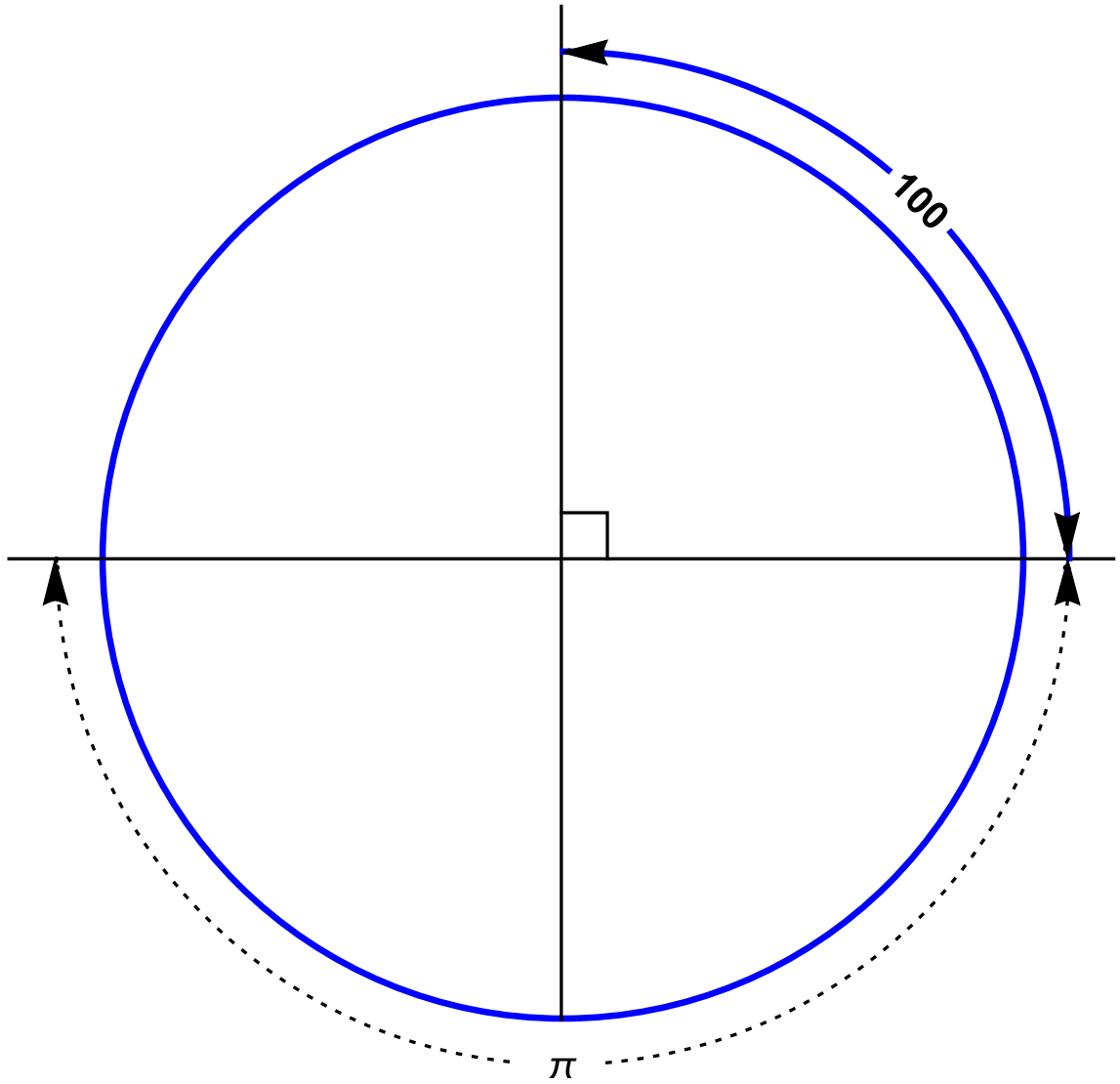
Euclid, 기원전 3세기





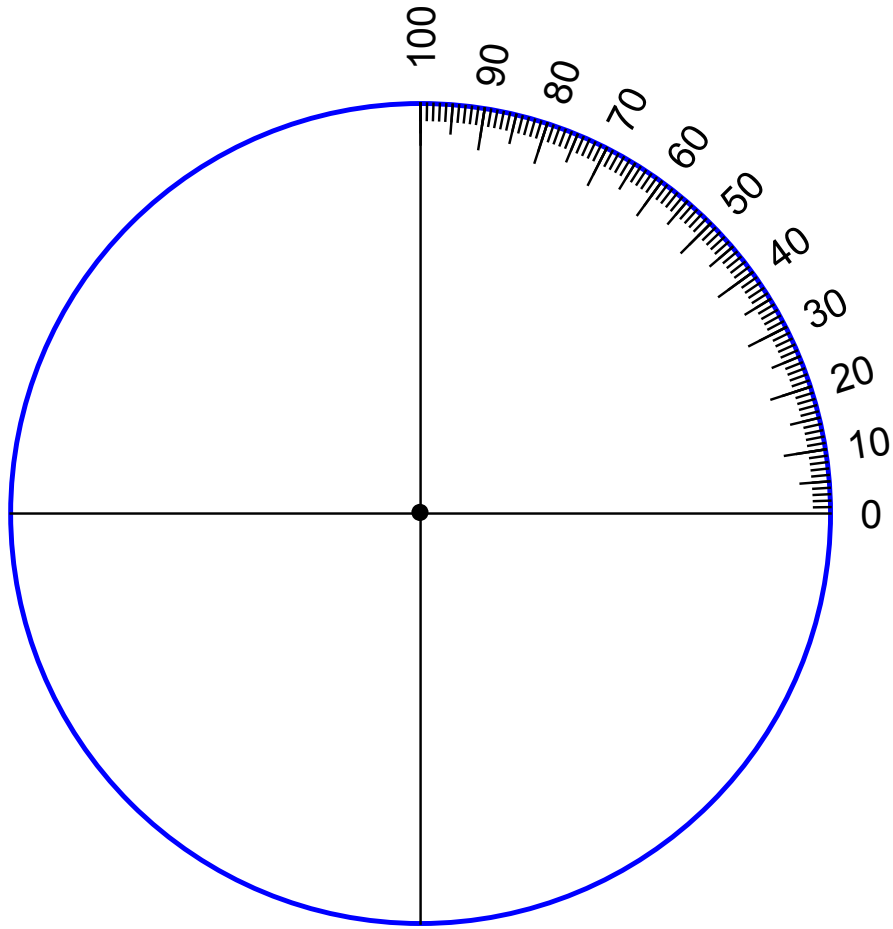
$$360^\circ = 4 \perp = 4 \angle R = \tau, \qquad \perp = \angle R = 90^\circ$$

French Revolution: $4 \angle R = 400^{\text{grad}}$



Q: 직각이나 평각이 각의 기준으로 적합한가?

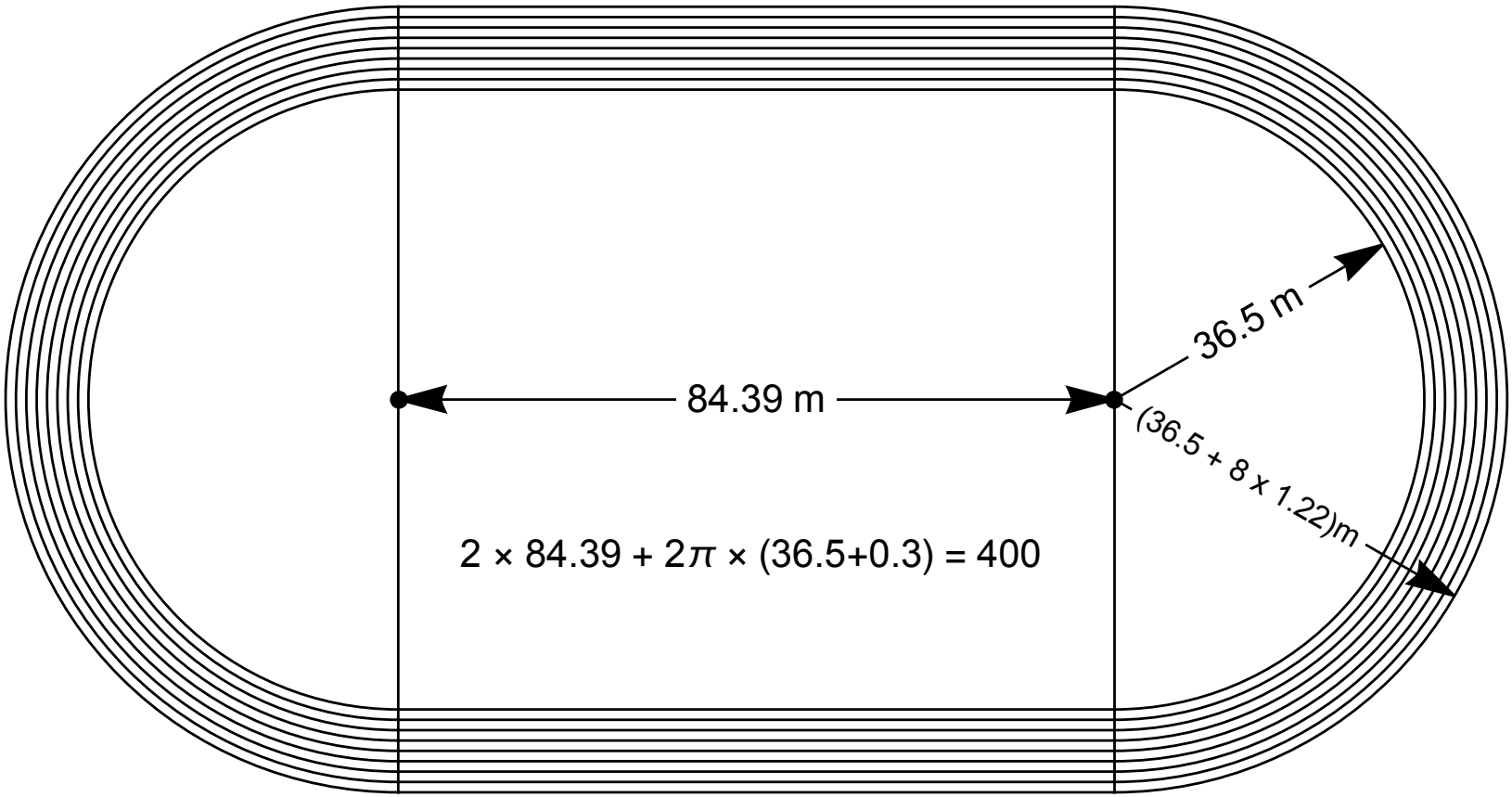
지구, 표준길이(1m), 십진법



$$\angle R = 100^{\mathsf{grad}} = 10^7\,\mathsf{m} \;= 10^4\;\;\mathsf{km}, \qquad 1^{\mathsf{grad}} = 100\,\mathsf{km}$$

π and the track

Why is the length of the standard international athletic track 400 m ?



1 mile $= 1609.344$ m, $\qquad \frac{1}{4}$ mile ≈ 400 m

Classes of functions

$$\mathcal{C}^0 = \{f \mid f \text{ is a continuous function}\}$$

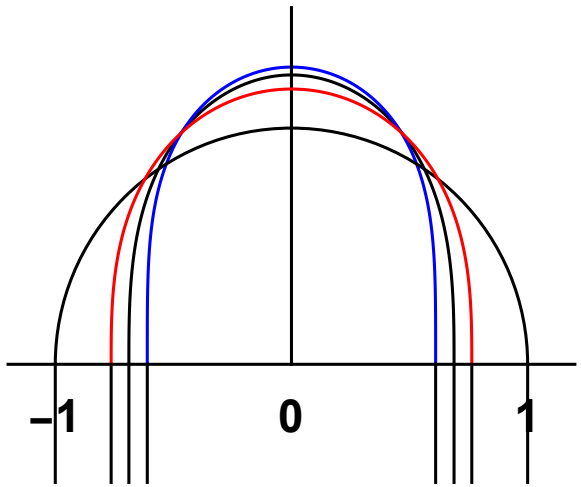
$$\mathcal{C}^1 = \{f \mid f' \in \mathcal{C}^0\}, \quad \mathcal{C}^2 = \{f \mid f' \in \mathcal{C}^1\}, \quad \dots$$

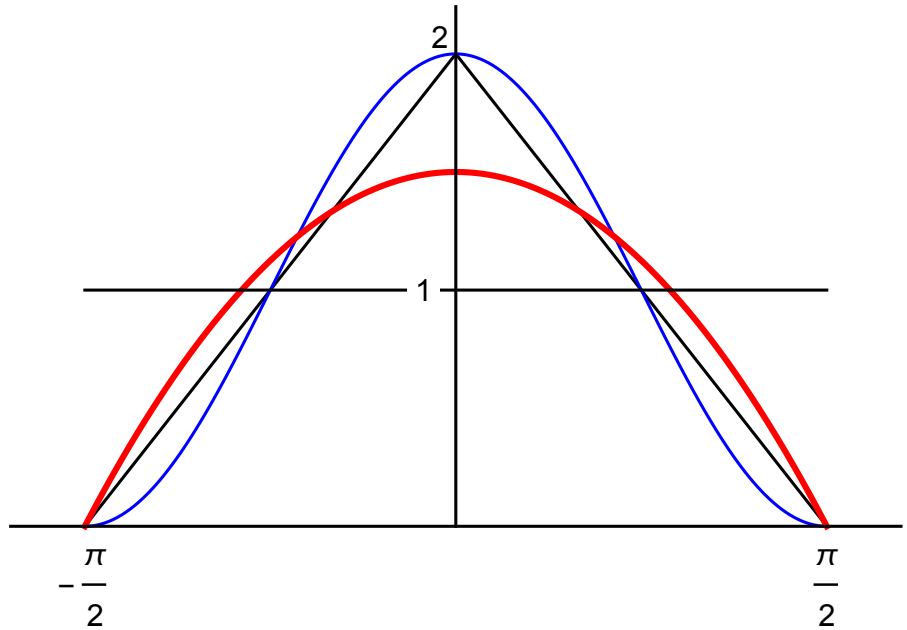
$$\mathcal{C}^{n+1} = \{f \mid f' \in \mathcal{C}^n\} \qquad (n = 0, 1, 2, \dots)$$

$$\mathcal{C}^0 \supset \mathcal{C}^1 \supset \boldsymbol{\mathcal{C}^2} \supset \cdots \supset \mathcal{C}^\infty \supset \mathcal{C}^\omega$$

Engineers must consider $\mathcal{C}^2$ construction.

$\mathcal{C}^1, \mathcal{C}^2, \mathcal{C}^3$ curves for track





Curves γ of the same length:

γ_0, $\mathcal{C}^1$, semicircle
γ_1 (red), γ_2, $\mathcal{C}^2$
γ_3 (blue), $\mathcal{C}^3$

Curvatures of γ:

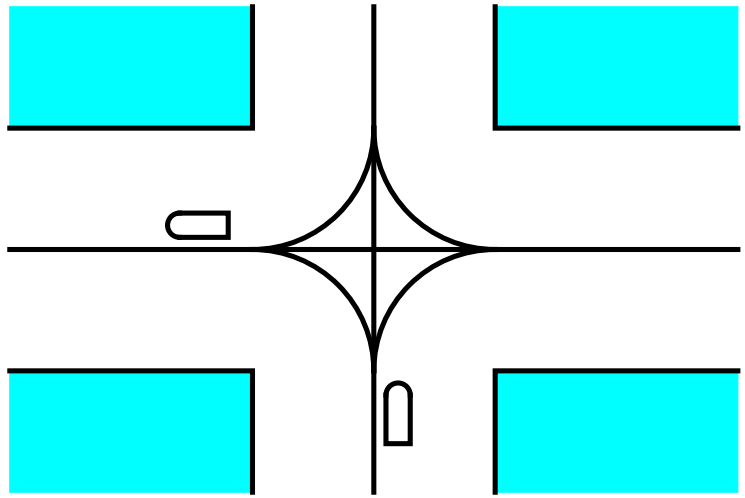
$$\kappa_0(s) = 1$$

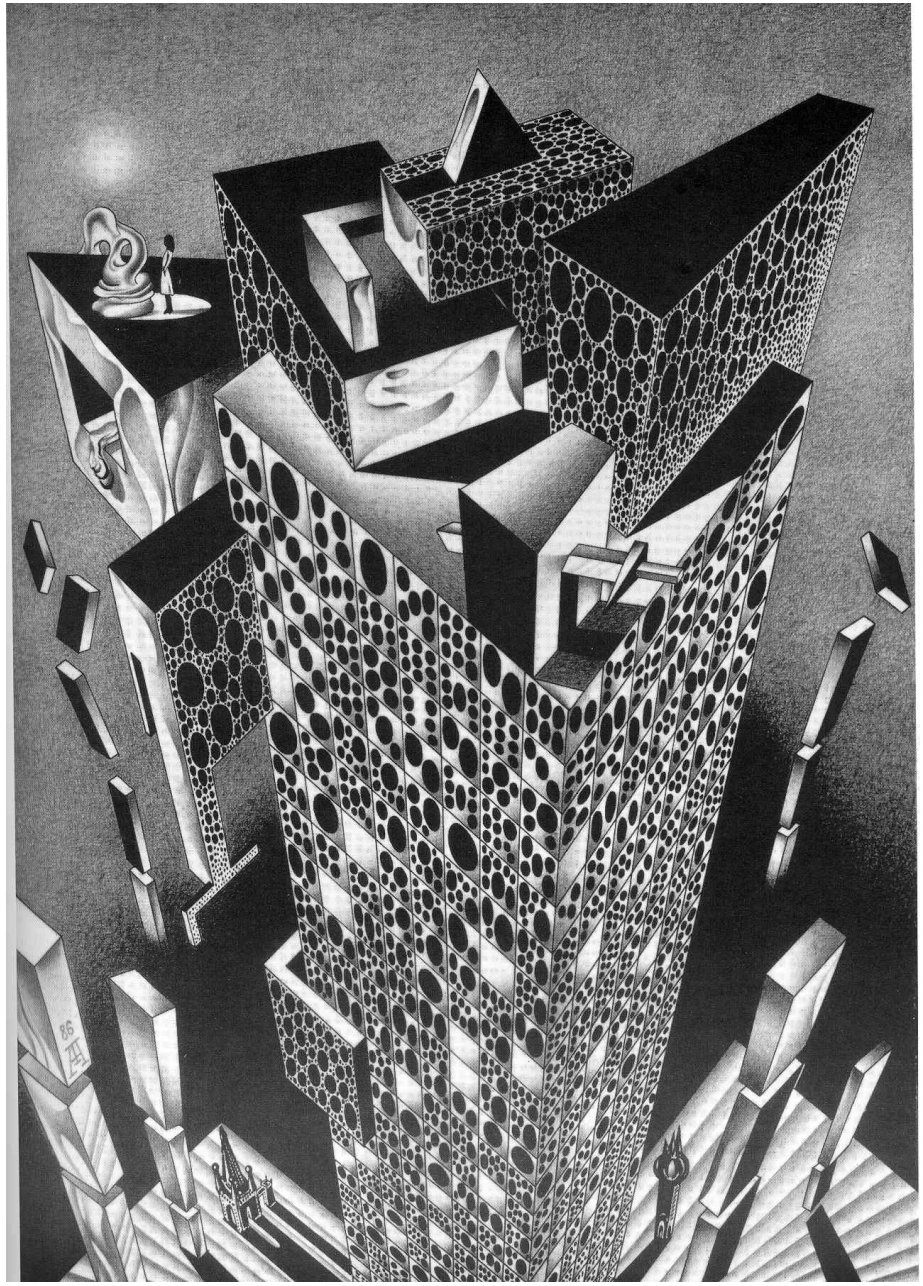
$$\kappa_1(s) = \tfrac{3}{2} - \tfrac{6}{\pi^2}s^2, \ \kappa_2(s) = 2 - \tfrac{4}{\pi}|s|$$

$$\kappa_3(s) = 1 + \cos 2s$$

The natural parameter $s \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ represents the arclength.
The integral of κ is π, the directional deviation of γ.

Traffic lanes and Roller coaster





π and e by Anatoly Fomenko (1945 –)