Towards a continuum limit from planar algebras.

Vaughan Jones,
Vanderbilt

August 11, 2014
Motivation.
Motivation.

Recent developments in subfactors, amplified by observations of Evans and Gannon, suggest that there are many more conformal field theories out there than one might have suspected.
Motivation.

Recent developments in subfactors, amplified by observations of Evans and Gannon, suggest that there are many more conformal field theories out there than one might have suspected.

Conformal field theory—a 2 dimensional quantum field theory with conformal symmetry so that its fields \( \Phi(z, \bar{z}) \) split as sums of tensor products of holomorphic fields, \( \Phi(z) \) and antiholomorphic ones.
Recent developments in subfactors, amplified by observations of Evans and Gannon, suggest that there are many more conformal field theories out there than one might have suspected.

Conformal field theory—a 2 dimensional quantum field theory with conformal symmetry so that its fields $\Phi(z, \bar{z})$ split as sums of tensor products of holomorphic fields, $\Phi(z)$ and antiholomorphic ones. The holomorphic theory is called a “chiral half” and is a one dimensional QFT in its own right.
Motivation.

Recent developments in subfactors, amplified by observations of Evans and Gannon, suggest that there are many more conformal field theories out there than one might have suspected. Conformal field theory—a 2-dimensional quantum field theory with conformal symmetry so that its fields $\Phi(z, \bar{z})$ split as sums of tensor products of holomorphic fields, $\Phi(z)$ and antiholomorphic ones. The holomorphic theory is called a “chiral half” and is a one-dimensional QFT in its own right. If its space-time is compactified to be a circle, the chiral theory exhibits diffeomorphism covariance implemented on the Lie algebra level by an action of the Virasoro algebra.
Recent developments in subfactors, amplified by observations of Evans and Gannon, suggest that there are many more conformal field theories out there than one might have suspected.

Conformal field theory—a 2 dimensional quantum field theory with conformal symmetry so that its fields $\Phi(z, \bar{z})$ split as sums of tensor products of holomorphic fields, $\Phi(z)$ and antiholomorphic ones. The holomorphic theory is called a “chiral half” and is a one dimensional QFT in its own right. If its space-time is compactified to be a circle, the chiral theory exhibits **diffeomorphism covariance implemented on the Lie algebra level by an action of the Virasoro algebra**. CFT’s arise as continuum limits of critical 2d statistical mechanical models such as the Ising and Potts models.
Motivation.

Recent developments in subfactors, amplified by observations of Evans and Gannon, suggest that there are many more conformal field theories out there than one might have suspected.

Conformal field theory-a 2 dimensional quantum field theory with conformal symmetry so that its fields $\Phi(z, \bar{z})$ split as sums of tensor products of holomorphic fields, $\Phi(z)$ and antiholomorphic ones. The holomorphic theory is called a “chiral half” and is a one dimensional QFT in its own right. If its space-time is compactified to be a circle, the chiral theory exhibits diffeomorphism covariance implemented on the Lie algebra level by an action of the Virasoro algebra. CFT’s arise as continuum limits of critical 2d statistical mechanical models such as the Ising and Potts models.
The n-point functions $\langle \Phi(z_1)\Phi(z_2)....\Phi(z_n) \rangle$ are singular if $z_i = z_j$ for some $i \neq j$ and satisfy differential equations whose monodromy give unitary representations of the braid groups which arise as the fundamental group of $\{z_1, z_2, ..., z_n | z_i \neq z_j \text{ for } i \neq j\}$. 
The n-point functions \( \langle \Phi(z_1)\Phi(z_2)\ldots\Phi(z_n) \rangle \) are singular if \( z_i = z_j \) for some \( i \neq j \) and satisfy differential equations whose monodromy give unitary representations of the braid groups which arise as the fundamental group of \( \{ z_1, z_2, \ldots, z_n \mid z_i \neq z_j \text{ for } i \neq j \} \). The simplest case of \( SU(2) \) WZW theory was calculated by Tsuchiya and Kanie who found that the braid group representations are those that arose in the theory of subfactors and gave the first of many knot polynomials and, in work of Witten, invariants of three manifolds.
The n-point functions $\langle \Phi(z_1)\Phi(z_2)....\Phi(z_n) \rangle$ are singular if $z_i = z_j$ for some $i \neq j$ and satisfy differential equations whose monodromy give unitary representations of the braid groups which arise as the fundamental group of $\{z_1, z_2, ..., z_n \mid z_i \neq z_j \text{ for } i \neq j\}$. The simplest case of $SU(2)$ WZW theory was calculated by Tsuchiya and Kanie who found that the braid group representations are those that arose in the theory of subfactors and gave the first of many knot polynomials and, in work of Witten, invariants of three manifolds.

The theory of vertex operator algebras (VOA’s) gives a mathematical version of one chiral half of a CFT directly via the fields which are called vertex operators.
The n-point functions $\langle \Phi(z_1) \Phi(z_2) \ldots \Phi(z_n) \rangle$ are singular if $z_i = z_j$ for some $i \neq j$ and satisfy differential equations whose monodromy give unitary representations of the braid groups which arise as the fundamental group of $\{z_1, z_2, \ldots, z_n | z_i \neq z_j \text{ for } i \neq j\}$. The simplest case of $SU(2)$ WZW theory was calculated by Tsuchiya and Kanie who found that the braid group representations are those that arose in the theory of subfactors and gave the first of many knot polynomials and, in work of Witten, invariants of three manifolds.

The theory of vertex operator algebras (VOA’s) gives a mathematical version of one chiral half of a CFT directly via the fields which are called vertex operators. The operator product expansion describes $\Phi_1(z) \Phi_2(w)$ as $z \to w$ and is the associativity structure of a VOA.
Haag and Kastler proposed an approach to quantum field theory in which von Neumann algebras $\mathcal{A}(\mathcal{O})$ are associated to all nice regions of space-time $\mathcal{O}$. $\mathcal{A}(\mathcal{O})$ is the algebra of all (bounded) observables localized in $\mathcal{O}$. 

Haag and Kastler proposed an approach to quantum field theory in which von Neumann algebras $\mathcal{A}(O)$ are associated to all nice regions of space-time $O$. $\mathcal{A}(O)$ is the algebra of all (bounded) observables localized in $O$. Physically natural axioms imply that the $\mathcal{A}(O)$ are type $\text{III}_1$ factors.
Haag and Kastler proposed an approach to quantum field theory in which von Neumann algebras $\mathcal{A}(\mathcal{O})$ are associated to all nice regions of space-time $\mathcal{O}$. $\mathcal{A}(\mathcal{O})$ is the algebra of all (bounded) observables localized in $\mathcal{O}$. Physically natural axioms imply that the $\mathcal{A}(\mathcal{O})$ are type $\text{III}_1$ factors.
A sub factor $N \subseteq M$ has an index $[M : N]$ which is a real number $\geq 1$ (or $\infty$). If the index is between 1 and 4 it is necessarily of the form $4 \cos^2 \frac{\pi}{n}$ for some $n \in \mathbb{Z}$, $n \geq 3$. 
A sub factor $N \subseteq M$ has an index $[M : N]$ which is a real number $\geq 1$ (or $\infty$). If the index is between 1 and 4 it is necessarily of the form $4 \cos^2 \frac{\pi}{n}$ for some $n \in \mathbb{Z}$, $n \geq 3$.

The fixed point algebra $M^G$ for an action of a finite group $G$ on $M$ is a very simple subfactor, of index $|G|$. 
A sub factor $N \subseteq M$ has an index $[M : N]$ which is a real number $\geq 1$ (or $\infty$). If the index is between 1 and 4 it is necessarily of the form $4 \cos^2 \pi/n$ for some $n \in \mathbb{Z}$, $n \geq 3$.

The fixed point algebra $M^G$ for an action of a finite group $G$ on $M$ is a very simple subfactor, of index $|G|$.

Finer than the index, subfactors have a principal graph analogous to the induction-restriction graph of a group and a subgroup. It is indeed the induction-restriction graph for $N - M$ and $N - N$ bimodules.
A sub factor $N \subseteq M$ has an index $[M : N]$ which is a real number $\geq 1$ (or $\infty$). If the index is between 1 and 4 it is necessarily of the form $4 \cos^2 \frac{\pi}{n}$ for some $n \in \mathbb{Z}, \ n \geq 3$.

The fixed point algebra $M^G$ for an action of a finite group $G$ on $M$ is a very simple subfactor, of index $|G|$.

Finer than the index, subfactors have a principal graph analogous to the induction-restriction graph of a group and a subgroup. It is indeed the induction-restriction graph for $N - M$ and $N - N$ bimodules.

If $[M : N] < 4$ the principal graph is of the form $A_n, D_{2n}, E_6$ or $E_8$ and these Coxeter graphs occur. $E_6$ and $E_8$ come in two complex conjugate versions (Ocneanu, unpublished).
A sub factor $N \subseteq M$ has an index $[M : N]$ which is a real number $\geq 1$ (or $\infty$). If the index is between 1 and 4 it is necessarily of the form $4 \cos^2 \pi/n$ for some $n \in \mathbb{Z}, \ n \geq 3$.

The fixed point algebra $M^G$ for an action of a finite group $G$ on $M$ is a very simple subfactor, of index $|G|$.

Finer than the index, subfactors have a principal graph analogous to the induction-restriction graph of a group and a subgroup. It is indeed the induction-restriction graph for $N - M$ and $N - N$ bimodules.

If $[M : N] < 4$ the principal graph is of the form $A_n, D_{2n}, E_6$ or $E_8$ and these Coxeter graphs occur. $E_6$ and $E_8$ come in two complex conjugate versions (Ocneanu, unpublished).

If this induction/restriction graph is finite the sub factor is said to be of finite depth, or “rational”.
There are at least three ways in which to obtain subfactors from CFT’s. Probably related but I don’t know of any general results.
There are at least three ways in which to obtain subfactors from CFT’s. Probably related but I don’t know of any general results.

1) From the braid group representations - consider the algebra generated by the infinite braid group - it’s a type II factor, and the algebra generated by the infinite braid group with vertical first string. Also a type II factor. The index would appear to be always finite for rational CFT. The $A_n$ principal graphs are obtainable in this way and many others.
There are at least three ways in which to obtain subfactors from CFT’s. Probably related but I don’t know of any general results.

1) From the braid group representations - consider the algebra generated by the infinite braid group - it’s a type II factor, and the algebra generated by the infinite braid group with vertical first string. Also a type II factor. The index would appear to be always finite for rational CFT. The $A_n$ principal graphs are obtainable in this way and many others.

2) Take the Boltzmann weights of a statistical mechanical model and use its site-to-site transfer matrix (or the nearest neighbor Hamiltonian in a quantum spin chain). Like for the braid group one may take the algebra generated by these local operators, and leave the first one out to obtain a subfactor.
There are at least three ways in which to obtain subfactors from CFT’s. Probably related but I don’t know of any general results.

1) From the braid group representations - consider the algebra generated by the infinite braid group - it’s a type II factor, and the algebra generated by the infinite braid group with vertical first string. Also a type II factor. The index would appear to be always finite for rational CFT. The $A_n$ principal graphs are obtainable in this way and many others.

2) Take the Boltzmann weights of a statistical mechanical model and use its site-to-site transfer matrix (or the nearest neighbor Hamiltonian in a quantum spin chain). Like for the braid group one may take the algebra generated by these local operators, and leave the first one out to obtain a subfactor. Can do such a thing for any subfactor but not clearly related to the CONTINUUM LIMIT.
There are at least three ways in which to obtain subfactors from CFT’s. Probably related but I don’t know of any general results.

1) From the braid group representations - consider the algebra generated by the infinite braid group - it’s a type II factor, and the algebra generated by the infinite braid group with vertical first string. Also a type II factor. The index would appear to be always finite for rational CFT. The $A_n$ principal graphs are obtainable in this way and many others.

2) Take the Boltzmann weights of a statistical mechanical model and use its site-to-site transfer matrix (or the nearest neighbor Hamiltonian in a quantum spin chain). Like for the braid group one may take the algebra generated by these local operators, and leave the first one out to obtain a subfactor. Can do such a thing for any subfactor but not clearly related to the CONTINUUM LIMIT.
3) The most interesting. Via the Doplicher-Haag-Roberts theory of superselection sectors. The “net” $O \to \mathcal{A}(O)$ may admit non-vacuum representations ("sectors") in which case the subfactor $\mathcal{A}(O) \subseteq \mathcal{A}(O')'$ (the primes denote causal complement and commutant) may be non-trivial.

$\mathcal{A}(O')': \mathcal{A}(O) \uparrow_2^1$ is called the statistical dimension of the sector.

For a chiral half of a CFT $O$ is an interval $I$ on the circle and $O'$ is the complementary interval.
3) The most interesting. Via the Doplicher-Haag-Roberts theory of superselection sectors. The “net” $\mathcal{O} \to \mathcal{A}(\mathcal{O})$ may admit non-vacuum representations (“sectors”) in which case the subfactor $\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{O}')'$ (the primes denote causal complement and commutant) may be non-trivial. Then $[\mathcal{A}(\mathcal{O}')' : \mathcal{A}(\mathcal{O})]^{1/2}$ is called the statistical dimension of the sector.
3) The most interesting. Via the Doplicher-Haag-Roberts theory of superselection sectors. The “net” \( \mathcal{O} \rightarrow \mathcal{A}(\mathcal{O}) \) may admit non-vacuum representations (“sectors”) in which case the subfactor \( \mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}(\mathcal{O}')' \) (the primes denote causal complement and commutant) may be non-trivial. Then \( [\mathcal{A}(\mathcal{O}'): \mathcal{A}(\mathcal{O})]^{1/2} \) is called the statistical dimension of the sector. For a chiral half of a CFT \( \mathcal{O} \) is an interval \( I \) on the circle and \( \mathcal{O}' \) is the complementary interval:
\[ U \langle I \rangle_2 \lesssim \langle I' \rangle_2 \]
$I' \subseteq \mathcal{O}(I) \subseteq \mathcal{O}'(I)$
These subfactors were calculated by Wassermann (unpublished) for $SU(N)$ WZW and further extended by Toledano, Loke, Xu, Longo, Kawahigashi, Carpi..... In particular they are all rational and all subfactors of index less than four occur.

In general the passage from a (unitary) VOA to a subfactor is to be achieved by realizing the fields as operator valued distributions on the circle, with $z$ being the circle variable. Smearing over functions supported in $I$ will generate the algebra $\mathfrak{A}(I)$. 
In the mid 90’s Haagerup discovered the rational subfactor of smallest index >4. Its principal graph is:

- index \( \frac{5 + \sqrt{13}}{2} \approx 4.30278 \)
- constructed in [Asaeda-Haagerup, math.OA/9803044]
Alternative constructions of the Haagerup were provided by Izumi, using Cuntz algebras, and Peters, using planar algebra.
Alternative constructions of the Haagerup were provided by Izumi, using Cuntz algebras, and Peters, using planar algebra. Haagerup’s discovery prompted two developments relevant to this talk:
(a) A systematic classification of all rational subfactors of index slightly larger than 4. (Morrison, Peters, Snyder et al)
Alternative constructions of the Haagerup were provided by Izumi, using Cuntz algebras, and Peters, using planar algebra. Haagerup’s discovery prompted two developments relevant to this talk:

(a) A systematic classification of all rational subfactors of index slightly larger than 4. (Morrison, Peters, Snyder et al)
(b) A search for the appearance of the Haagerup, and any other new discoveries, in CFT.
Alternative constructions of the Haagerup were provided by Izumi, using Cuntz algebras, and Peters, using planar algebra. Haagerup’s discovery prompted two developments relevant to this talk:
(a) A systematic classification of all rational subfactors of index slightly larger than 4. (Morrison, Peters, Snyder et al)
(b) A search for the appearance of the Haagerup, and any other new discoveries, in CFT.
The project (a) has been carried out up to index $3 + \sqrt{5}$. Here is the resulting “map” of subfactors (Morrison):
Recent developments in subfactors

Towards a continuum limit from planar algebras.

August 11, 2014
Possibly because of accidents of small index, many of the principal graphs begin with a long chunk of $A_n$ graph. The length of this chunk is called the supertransitivity because of a compelling analogy with the transitivity of group actions.
Possibly because of accidents of small index, many of the principal graphs begin with a long chunk of $A_n$ graph. The length of this chunk is called the supertransitivity because of a compelling analogy with the transitivity of group actions. High supertransitivity means that there is nothing to see or use in the subfactor until it has been explored in considerable depth. This creates difficulties.
Possibly because of accidents of small index, many of the principal graphs begin with a long chunk of $A_n$ graph. The length of this chunk is called the supertransitivity because of a compelling analogy with the transitivity of group actions. High supertransitivity means that there is nothing to see or use in the subfactor until it has been explored in considerable depth. This creates difficulties.

It is a fascinating open question as to whether supertransitivity is bounded for index $>4$. If this is the case the situation is quite analogous to that of group actions where high transitivity implies either the symmetric or alternating groups, or the finite family of Mathieu groups.
The search for the Haagerup in CFT.

The Haagerup subfactor cannot occur as $\mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}(\mathcal{O}')'$. This is because it has non-commutative fusion rules and the fusion in CFT is braided, hence commutative.

But one should not give up. One may take Ocneanu’s asymptotic inclusion (unpublished) of the subfactor which corresponds to the Drinfeld center of the fusion category, which is braided, hence commutative.

Evans and Gannon have observed that the S-matrix of this modular tensor category can be understood in terms of modular forms as if the category were some kind of conglomerate of the representation theory of $S_3$ and $SO(13)$. They speculate that there is a specific VOA for this CFT and give precise properties that it must have.

Izumi’s Cuntz algebra method generalizes and several of the subfactors in the map can be obtained in this way. The speculations of Evans and Gannon seem to apply to these generalizations as well.

Vaughan Jones, Vanderbilt
The search for the Haagerup in CFT.

The Haagerup subfactor cannot occur as \( \mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{O}')' \). This is because it has non-commutative fusion rules and the fusion in CFT is braided, hence commutative. But one should not give up. One may take Ocneanu’s asymptotic inclusion (unpublished) of the subfactor which corresponds to the Drinfeld center of the fusion category, which is braided, hence commutative.
The search for the Haagerup in CFT.

The Haagerup subfactor cannot occur as $\mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}(\mathcal{O}')'$. This is because it has non-commutative fusion rules and the fusion in CFT is braided, hence commutative. But one should not give up. One may take Ocneanu’s asymptotic inclusion (unpublished) of the subfactor which corresponds to the Drinfeld center of the fusion category, which is braided, hence commutative. Evans and Gannon have observed that the S-matrix of this modular tensor category can be understood in terms of modular forms as if the category were some kind of conglomerate of the representation theory of $S_3$ and $SO(13)$. 
An aside on super transitivity.

The search for the Haagerup in CFT.

The Haagerup subfactor cannot occur as $\mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}(\mathcal{O}')'$. This is because it has non-commutative fusion rules and the fusion in CFT is braided, hence commutative. But one should not give up. One may take Ocneanu’s asymptotic inclusion (unpublished) of the subfactor which corresponds to the Drinfeld center of the fusion category, which is braided, hence commutative. Evans and Gannon have observed that the S-matrix of this modular tensor category can be understood in terms of modular forms as if the category were some kind of conglomerate of the representation theory of $S_3$ and $SO(13)$. They speculate that there is a specific VOA for this CFT and give precise properties that it must have.
The search for the Haagerup in CFT.

The Haagerup subfactor cannot occur as $\mathcal{A}(\mathcal{O}) \subseteq \mathcal{A}(\mathcal{O}')'$. This is because it has non-commutative fusion rules and the fusion in CFT is braided, hence commutative. But one should not give up. One may take Ocneanu’s asymptotic inclusion (unpublished) of the subfactor which corresponds to the Drinfeld center of the fusion category, which is braided, hence commutative. Evans and Gannon have observed that the S-matrix of this modular tensor category can be understood in terms of modular forms as if the category were some kind of conglomerate of the representation theory of $S_3$ and $SO(13)$. They speculate that there is a specific VOA for this CFT and give precise properties that it must have.

Izumi’s Cuntz algebra method generalizes and several of the subfactors in the map can be obtained in this way. The speculations of Evans and Gannon seem to apply to these generalizations as well.
So it is possible that all of the “exotica” so far produced might be obtainable in some useful sense from conformal field theory.
An aside on super transitivity.

So it is possible that all of the “exotica” so far produced might be obtainable in some useful sense from conformal field theory. One must therefore confront the question— is it possible to construct a CFT directly from the data of a subfactor? (At least a rational one.)
So it is possible that all of the “exotica” so far produced might be obtainable in some useful sense from conformal field theory. One must therefore confront the question— is it possible to construct a CFT directly from the data of a subfactor? (At least a rational one.) This was actually one of the motivations of my construction of “planar algebras” which to my surprise turned out to be little different from Kuperberg’s spiders.
So it is possible that all of the “exotica” so far produced might be obtainable in some useful sense from conformal field theory. One must therefore confront the question- is it possible to construct a CFT directly from the data of a subfactor? (At least a rational one.) This was actually one of the motivations of my construction of “planar algebras” which to my surprise turned out to be little different from Kuperberg’s spiders. Planar algebras are graded vector spaces with canonical inner products in each grading so one might try to construct a VOA directly since it has similar ingredients.
So it is possible that all of the “exotica” so far produced might be obtainable in some useful sense from conformal field theory. One must therefore confront the question- is it possible to construct a CFT directly from the data of a subfactor? (At least a rational one.) This was actually one of the motivations of my construction of “planar algebras” which to my surprise turned out to be little different from Kuperberg’s spiders. Planar algebras are graded vector spaces with canonical inner products in each grading so one might try to construct a VOA directly since it has similar ingredients. It would be surprising if this naive construction worked.
An aside on super transitivity.

So it is possible that all of the “exotica” so far produced might be obtainable in some useful sense from conformal field theory. One must therefore confront the question- is it possible to construct a CFT directly from the data of a subfactor? (At least a rational one.) This was actually one of the motivations of my construction of “planar algebras” which to my surprise turned out to be little different from Kuperberg’s spiders. Planar algebras are graded vector spaces with canonical inner products in each grading so one might try to construct a VOA directly since it has similar ingredients. It would be surprising if this naive construction worked. But not out of the question as one would still face the tough question of identifying the subfactors of the corresponding CFT.
An aside on super transitivity.

So it is possible that all of the “exotica” so far produced might be obtainable in some useful sense from conformal field theory. One must therefore confront the question— is it possible to construct a CFT directly from the data of a subfactor? (At least a rational one.) This was actually one of the motivations of my construction of “planar algebras” which to my surprise turned out to be little different from Kuperberg’s spiders. Planar algebras are graded vector spaces with canonical inner products in each grading so one might try to construct a VOA directly since it has similar ingredients. It would be surprising if this naive construction worked. But not out of the question as one would still face the tough question of identifying the subfactors of the corresponding CFT.

The “royal road” would be to use some elements of the planar algebra as Boltzmann weights for a critical stat mech model, take the continuum limit and get a subfactor from one of the chiral halves.
So it is possible that all of the “exotica” so far produced might be obtainable in some useful sense from conformal field theory. One must therefore confront the question— is it possible to construct a CFT directly from the data of a subfactor? (At least a rational one.) This was actually one of the motivations of my construction of “planar algebras” which to my surprise turned out to be little different from Kuperberg’s spiders. Planar algebras are graded vector spaces with canonical inner products in each grading so one might try to construct a VOA directly since it has similar ingredients. It would be surprising if this naive construction worked. But not out of the question as one would still face the tough question of identifying the subfactors of the corresponding CFT. The “royal road” would be to use some elements of the planar algebra as Boltzmann weights for a critical stat mech model, take the continuum limit and get a subfactor from one of the chiral halves. Then identify that subfactor with the original one.
An aside on super transitivity.

So it is possible that all of the “exotica” so far produced might be obtainable in some useful sense from conformal field theory. One must therefore confront the question— is it possible to construct a CFT directly from the data of a subfactor? (At least a rational one.) This was actually one of the motivations of my construction of “planar algebras” which to my surprise turned out to be little different from Kuperberg’s spiders. Planar algebras are graded vector spaces with canonical inner products in each grading so one might try to construct a VOA directly since it has similar ingredients. It would be surprising if this naive construction worked. But not out of the question as one would still face the tough question of identifying the subfactors of the corresponding CFT. The “royal road” would be to use some elements of the planar algebra as Boltzmann weights for a critical stat mech model, take the continuum limit and get a subfactor from one of the chiral halves. Then identify that subfactor with the original one. The royal road is paved with mathematical difficulties. Even in the cases where it “should” work this would amount to solving major mathematical difficulties of the continuum limit.
The most direct approach would be to construct a net of type $\text{III}_1$ factors with diffeomorphism covariance on the circle directly from the planar algebra/subfactor.
An aside on super transitivity.

The most direct approach would be to construct a net of type III$_1$ factors with diffeomorphism covariance on the circle directly from the planar algebra/subfactor.

It is possible to construct a toy theory from a planar algebra/subfactor. The annular representation category has a “vacuum” representation and a family of other representations with a fusion.
The most direct approach would be to construct a net of type III$_1$ factors with diffeomorphism covariance on the circle directly from the planar algebra/subfactor. It is possible to construct a toy theory from a planar algebra/subfactor. The annular representation category has a “vacuum” representation and a family of other representations with a fusion. One can form a Hilbert space as a direct limit for the directed set of finite subsets of the circle. One obtains von Neumann algebras $\mathcal{A}(I)$ with some of the properties of a CFT net, in particular causality: $\mathcal{A}(I) \subseteq \mathcal{A}(I')'$. But diffeomorphism covariance is present but not continuous so the local algebras are not factors, and not of type III.
The most direct approach would be to construct a net of type III$_1$ factors with diffeomorphism covariance on the circle directly from the planar algebra/subfactor.

It is possible to construct a toy theory from a planar algebra/subfactor. The annular representation category has a “vacuum” representation and a family of other representations with a fusion. One can form a Hilbert space as a direct limit for the directed set of finite subsets of the circle. One obtains von Neumann algebras $\mathcal{A}(I)$ with some of the properties of a CFT net, in particular causality: $\mathcal{A}(I) \subseteq \mathcal{A}(I')'$. But diffeomorphism covariance is present but not continuous so the local algebras are not factors, and not of type III. Another toy approach is to consider the set of dyadic rationals on the circle. In this way one obtains a theory where the Thompson groups play the role of the diffeomorphism groups.
An aside on super transitivity.

The most direct approach would be to construct a net of type $\text{III}_1$ factors with diffeomorphism covariance on the circle directly from the planar algebra/subfactor. It is possible to construct a toy theory from a planar algebra/subfactor. The annular representation category has a “vacuum” representation and a family of other representations with a fusion. One can form a Hilbert space as a direct limit for the directed set of finite subsets of the circle. One obtains von Neumann algebras $\mathcal{A}(I)$ with some of the properties of a CFT net, in particular causality: $\mathcal{A}(I) \subseteq \mathcal{A}(I')'$. But diffeomorphism covariance is present but not continuous so the local algebras are not factors, and not of type $\text{III}$. Another toy approach is to consider the set of dyadic rationals on the circle. In this way one obtains a theory where the Thompson groups play the role of the diffeomorphism groups. The ensuing unitary projective representations of the Thompson groups are not entirely without interest. It is to be hoped that by including more data from the subfactor one can obtain a genuine net of local observables in this way.
appendix: how to construct the Thompson group representations.
appendix: how to construct the Thompson group representations. The idea is to take an annular representation of the planar algebra which consists of vector spaces $V_n$ for every non-negative integer $n$ and embed them inductively into one another using annular tangles from the planar algebra.
appendix: how to construct the Thompson group representations. The idea is to take an annular representation of the planar algebra which consists of vector spaces $V_n$ for every non-negative integer $n$ and embed them inductively into one another using annular tangles from the planar algebra. The simplest way to do this is to start with a planar algebra 1-box $R$ (an element corresponding to a single boundary point). Then given any finite subsets $F \subseteq G$ of the circle one may embed $V_F$ (which is just a copy of $V_{|F|}$) into $V_G$ via the following annular tangle:
appendix: how to construct the Thompson group representations. The idea is to take an annular representation of the planar algebra which consists of vector spaces $V_n$ for every non-negative integer $n$ and embed them inductively into one another using annular tangles from the planar algebra. The simplest way to do this is to start with a planar algebra 1-box $R$ (an element corresponding to a single boundary point). Then given any finite subsets $F \subseteq G$ of the circle one may embed $V_F$ (which is just a copy of $V_{|F|}$ into $V_G$ via the following annular tangle:
Red points = \( F \)
Blue points = points in \( G \) not in \( F \)
The Hilbert space of the theory is then the direct limit of the Hilbert spaces $V_F$. 
The Hilbert space of the theory is then the direct limit of the Hilbert spaces $V_F$.
The diffeomorphism group acts in an obvious way.
The Hilbert space of the theory is then the direct limit of the Hilbert spaces $V_F$.
The diffeomorphism group acts in an obvious way. But things are highly non-separable/discontinuous. To get separability reduce to, e.g. dyadic rationals.
An aside on super transitivity.

The Hilbert space of the theory is then the direct limit of the Hilbert spaces $V_F$.
The diffeomorphism group acts in an obvious way. But things are highly non-separable/discontinuous. To get separability reduce to, e.g. dyadic rationals. Then only the Thompson group $V$ of piecewise linear homeomorphisms will act, with the Thompson group $F$ being the local "diffeomorphism group".

By a standard uncappability argument one can show that all irreducible reps of $F$ thus obtained are those on $\ell^2$(finite sets of dyadic rationals). Local algebras $A(I)$ can be obtained by looking at the dyadic points in $I$. Much more interesting representations can be obtained by using a 4-box $R$ to embed $V^2_n$ in $V^2_{n+1}$: (restrict to the interval case)
The Hilbert space of the theory is then the direct limit of the Hilbert spaces $V_F$.
The diffeomorphism group acts in an obvious way. But things are highly non-separable/discontinuous. To get separability reduce to, e.g. dyadic rationals. Then only the Thompson group $V$ of piecewise linear homeomorphisms will act, with the Thompson group $F$ being the local "diffeomorphism group". By a standard uncappability argument one can show that all irreducible reps of $F$ thus obtained are those on $\ell^2$(finite sets of dyadic rationals).
The Hilbert space of the theory is then the direct limit of the Hilbert spaces $V_F$.

The diffeomorphism group acts in an obvious way. But things are highly non-separable/discontinuous. To get separability reduce to, e.g. dyadic rationals. Then only the Thompson group $V$ of piecewise linear homeomorphisms will act, with the Thompson group $F$ being the local "diffeomorphism group". By a standard uncappability argument one can show that all irreducible reps of $F$ thus obtained are those on $\ell^2$ (finite sets of dyadic rationals). Local algebras $\mathcal{A}(I)$ can be obtained by looking at the dyadic points in $I$.

Much more interesting representations can be obtained by using a 4-box $R$ to embed $V_{2^n}$ in $V_{2^{n+1}}$:
The Hilbert space of the theory is then the direct limit of the Hilbert spaces $V_F$.

The diffeomorphism group acts in an obvious way. But things are highly non-separable/discontinuous. To get separability reduce to, e.g. dyadic rationals. Then only the Thompson group $V$ of piecewise linear homeomorphisms will act, with the Thompson group $F$ being the local "diffeomorphism group". By a standard uncappability argument one can show that all irreducible reps of $F$ thus obtained are those on $\ell^2$ (finite sets of dyadic rationals). Local algebras $\mathcal{A}(I)$ can be obtained by looking at the dyadic points in $I$.

Much more interesting representations can be obtained by using a 4-box $R$ to embed $V_{2^n}$ in $V_{2^{n+1}}$: (restrict to the interval case)
An aside on super transitivity.
The Thompson group action is (well) defined by stabilizing using the embeddings. Hence the Thompson groups act unitarily on the direct limit Hilbert space.
The Thompson group action is (well) defined by stabilizing using the embeddings. Hence the Thompson groups act unitarily on the direct limit Hilbert space. The coefficients of the representation are partition functions for models on the Bethe lattice whose Boltzmann weights are given by the element \( R \) in a particular planar algebra. The Bethe lattice is a simplistic, but used, stat mech model which tends to be *solvable* hence there is some hope for serious progress in studying these representations.
The Thompson group action is (well) defined by stabilizing using the embeddings. Hence the Thompson groups act unitarily on the direct limit Hilbert space. The coefficients of the representation are partition functions for models on the Bethe lattice whose Boltzmann weights are given by the element $R$ in a particular planar algebra. The Bethe lattice is a simplistic, but used, stat mech model which tends to be solvable hence there is some hope for serious progress in studying these representations. $R$ could be chosen to be a crossing from knot theory. In this way one obtains knots and links from the Thompson group, and, by varying $R$, sequences of unitary representations tending to the trivial and tending to the left regular representation.