Parrondo games with spatial dependence and a related spin system

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Abstract

Toral introduced so-called cooperative Parrondo games, in which there are $N \geq 3$ players arranged in a circle. At each turn one player is randomly chosen to play. He plays either game A or game B, depending on the strategy. Game A results in a win or loss of one unit based on the toss of a fair coin. Game B results in a win or loss of one unit based on the toss of a biased coin, with the amount of the bias depending on whether none, one, or two of the player's two nearest neighbors have won their most recent games. Game A is fair, so the games are said to exhibit the Parrondo effect if game B is losing or fair and the random mixture $C := (1/2)(A + B)$ is winning. With μ_B^N (resp., μ_C^N) denoting the mean profit per turn to the ensemble of N players always playing game B (resp., C), we give sufficient conditions for $\lim_{N\to\infty} \mu_B^N$ to exist and show that $\lim_{N\to\infty} \mu_C^N$ nearly always exists, with the limits expressible in terms of a parameterized spin system on the one-dimensional integer lattice. For a particular choice of the parameters, we show that the Parrondo effect (i.e., $\mu_B^N \leq 0$ and $\mu_C^N > 0$) is present in the N-player model if and only if N is even. For the same choice of the parameters, we show that, with a suitable interpretation and for certain initial distributions, the Parrondo effect is present in the spin system if and only if N is even, N being the number of consecutive players whose collective profit is tracked.