A generalization of littlewood-paley inequality for the fractional Laplacian and applications to SPDEs

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Abstract

For several decades, partial differential equations with the fractional Laplacian have been studied by many authors. Motivated by this, we were interested in constructing an L_p-theory of stochastic partial differential equations of the type

$$du = -(-\Delta)^{\alpha/2} u \, dt + \sum_{k=1}^{\infty} f^k dw_t^k, \quad u(0,x) = 0. \tag{0.1}$$

Under an appropriate condition, the solution of this problem is given by

$$u(t,x) = \sum_{k=1}^{\infty} \int_0^t T_{\alpha,t-s} f^k(s,\cdot)(x) dw_s^k,$$

where $p(t,x) := \int_{\mathbb{R}^d} e^{i\xi\cdot x} e^{-t|\xi|^\alpha} d\xi$ and $T_{\alpha,t} f(x) := (p(t,\cdot) * f(\cdot))(x) := \int_{\mathbb{R}^d} p(t,x-y) f(y) dy$.

By Burkholder-Davis-Gundy inequality,

$$\mathbb{E} \int_0^T \|\partial_x^{\alpha/2} u(t,\cdot)\|_{L_p}^p dt \le N(p) \, \mathbb{E} \int_0^T \int_{\mathbb{R}^d} \left[\int_0^t |\partial_x^{\alpha/2} T_{\alpha,t-s} f(s,\cdot)(x)|_{\ell_2}^2 ds \right]^{p/2} dxdt.$$

So one needs to estimate

$$\int_0^T \int_{\mathbb{R}^d} \left[\int_0^t |\partial_x^{\alpha/2} T_{\alpha,t-s} f(s,\cdot)(x)|_{\ell_2}^2 ds \right]^{p/2} dxdt$$

to get a L_p-estimation of the solution of (0.1).

In this talk, we introduce some related results to this work and how to get

$$\int_0^T \int_{\mathbb{R}^d} \left[\int_0^t |\partial_x^{\alpha/2} T_{\alpha,t-s} f(s,\cdot)(x)|_{\ell_2}^2 ds \right]^{p/2} dxdt \le N(\alpha,p,T) \mathbb{E} \int_0^T \| |f|_{\ell_2} \|_{L_p}^p ds.$$