Random walk, quantum walk, and quantum probability

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Abstract

Quantum probability is a non-commutative extension of classical probability. In stead of probability measure space quantum probability is based on a pair $(\mathcal{A}, \phi)$ of a $*$-algebra $\mathcal{A}$ and a state ϕ, which is called an algebraic probability space. For a classical probability space $(\Omega, \mathcal{F}, P)$, letting $\mathcal{A}$ be the space of random variables having moments of all orders and setting $\phi(X) = \mathbf{E}[X]$, we obtain an algebraic probability space, where a random variable X is studied by means of quantum probabilistic techniques. In this talk we introduce some of the most basic concepts of quantum probability, e.g., quantum decomposition, various concepts of independence, quantum central limit theorems, and so forth. Then we discuss how quantum probability is applied for the study of random walks and quantum walks on graphs, in particular for their spectral analysis. The former is a well-known subject in classical probability, while the latter is relatively new and viewed as a quantum mechanical extension of a random walk. We expect that quantum probability is useful also for quantum walks.