

# Recent Developments in Quantum White Noise Calculus

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## Abstract

On the basis of the quantum white noise theory we introduce the notion of quantum white noise derivatives of Fock space operators and study the basic properties of the derivatives. Then the quantum white noise derivatives become derivations with respect to the Wick product. The Wick derivation property of the quantum white noise derivatives is successfully applied for the study of implementation problem for the canonical commutation relation of which the solution suggests a kind of general transformation involving Fourier–Gauss and Fourier–Mehler transform, Bogoliubov transformation and a non-commutative extension of the Girsanov transformation. Finally, by applying complex white noise, we study normal ordered forms of Fock space operators and as an application of the normal order forms we study the unitary property of the implementations of the canonical commutation relations.

## References

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