Ninth Bielefeld-SNU Joint Workshop in Mathematics
Abstracts for invited talks

Self-adjoint Laplacians, Symmetric Semigroups and Diffusions on Hyperbolic Attractors

Shayan Alikhanloo

Analysis on smooth manifolds, foliated spaces and fractals in terms of Dirichlet forms is well established. But such an analysis on hyperbolic attractors is yet to be explored. We use the core material and central results from the theory of hyperbolic dynamical systems such as the stable manifold theorem and physical measures to introduce self-adjoint Laplacians, symmetric Markov semigroups and symmetric diffusions via Dirichlet forms. In particular, this may be seen as far-reaching extension of well-known classical analysis on geodesic flows on manifolds of negative sectional curvature. This is joint work with Michael Hinz (Bielefeld).

On the characteristic polynomial of the eigenvalue moduli of random normal matrices

Sungsoo Byun

In this talk, I will discuss the characteristic polynomial of the eigenvalue moduli drawn from the Mittag-Leffler ensemble, a two-dimensional determinantal point process that generalises the Ginibre point process. I will present precise asymptotic behaviours of the moment generating function, which involves a large structured determinant whose weight is supported on the whole complex plane, is rotation-invariant, and has both jump- and root-type singularities. In particular, I will explain that such asymptotic behaviours surprisingly yield a new kind of ingredient called associated Hermite polynomials. This is based on joint work with Christophe Charlier.

An algebraic approach for the weak coupling of multiple Lohe tensor models

Seung-Yeal Ha

In this talk, we present a systematic algebraic approach for the weak coupling of Cauchy problems to multiple Lohe tensor models. For this, we identify an admissible Cauchy problem to the Lohe tensor (LT) model with a characteristic symbol consisting of four tuples in terms of a size vector, a natural frequency tensor, a coupling strength tensor and admissible initial configuration. In this way, the collection of all admissible Cauchy problems to the LT models is equivalent to the space of characteristic symbols. On the other hand, we introduce a binary operation, namely "fusion operation" as a binary operation between characteristic symbols. It turns out that the fusion operation satisfies an associativity and admits the identity element in the space of characteristic symbols which naturally forms a monoid. By virtue of the fusion operation, the weakly coupled system of multi tensor models can be obtained by applying the fusion operation of multiple characteristic symbols corresponding to the Lohe tensor models. As a concrete example, we consider a weak coupling of the swarm sphere model and the Lohe matrix model, and provide sufficient framework leading to emergent dynamics to the proposed weakly coupled model. This is a joint work with Dohyun Kim (Sungshin Women's Univ.) and Hansol Park (Simon Fraser Univ.).

New Developments of solving PDE using Neural Networks

Myungjoo Kang

Following the success in machine learning tasks, neural network (NN) based methods are starting to show promise results in classical problems of applied mathematics. In this talk, we introduce two important recent approaches for solving partial differential equations: physics-informed neural networks (PINNs) and neural operators. PINNs employ NNs as approximators for the solution function to a PDE. Alternatively, neural operators aim at use NNs as the ansatz of the solution operator for a family of PDEs. Specifically, the neural operator generalizes the conventional NNs to the operator between infinite-dimensional function spaces. We contrast their ability and suitability for complex PDEs, and then introduce various applications.

Pointwise inequalities for solutions to the fractional heat equation

Moritz Kaßmann

We study inequalities for nonnegative solutions to the fractional heat equation and related equations. In particular, we establish a differential Li-Yau inequality that implies the well-known parabolic Harnack inequality. We transfer these results to the case of bounded domains. Moreover, we compare solutions to the fractional heat equation with solutions to the corresponding degenerate elliptic equation.The talk is based on a joint work with Tuhin Ghosh.

The Wiener criterion for nonlocal Dirichlet problems

Minhyun Kim

In this talk, we study the boundary behavior of solutions to the Dirichlet problems for integro-differential operators with order of differentiability $s \in (0,1)$ and summability $p > 1$. We establish a nonlocal counterpart of the Wiener criterion, which characterizes a regular boundary point in terms of the nonlocal nonlinear potential theory.

Sticky-reflected stochastic heat equation driven by colored noise

Vitalii Konarovskyi

We will discuss the existence of a sticky-reflected solution to the heat equation on the space interval [0,1] driven by colored noise. The process can be interpreted as an infinite-dimensional analog of the sticky-reflected Brownian motion on the real line but, in this case, the solution obeys the ordinary stochastic heat equation, except the points where it reaches zero. The solution has no noise at zero and a drift pushes it to stay positive. The proof of the existence is based on a new approach that can be applied to some other types of SPDEs with discontinuous coefficients.

Simple random walk on a long-range percolation cluster: heat kernel bounds and spectral dimension

Takashi Kumagai

Consider the long-range percolation model on the integer lattice $\mathbb{Z}^d$ in which all nearest-neighbour edges are present and otherwise x and y are connected with probability $q_{x,y} := 1-\exp(-|x-y|^{-s})$, independently of the state of other edges. Throughout the regime where the model yields a locally-finite graph, (i.e. for $s > d$,) we determine the spectral dimension of the associated simple random walk, apart from at the exceptional value $d = 1, s = 2$, where the spectral dimension is discontinuous. Towards this end, we present various on-diagonal heat kernel bounds, a number of which are new. In particular, the lower bounds are derived through the application of a general technique that utilizes the translation invariance of the model. We highlight that, applying this general technique, we are able to partially extend our main result beyond the nearest-neighbor setting, and establish lower heat kernel bounds over the range of parameters $s \in (d, 2d)$. Our approach is applicable to short-range models as well. This is a joint work with Van Hao Can (Hanoi) and David A. Croydon (Kyoto).

Existence and uniqueness of (infinitesimally) invariant measures for second order partial differential operators on Euclidean space

Gerald Trutnau

We consider a locally uniformly strictly elliptic second order partial differential operator in $\mathbb{R}^d$, $d \geq 2$, with low regularity assumptions on its coefficients, as well as an associated Hunt process and semigroup. The Hunt process is known to solve a corresponding stochastic differential equation that is pathwise unique. In this situation, we study the relation of invariance, infinitesimal invariance, recurrence, transience, conservativeness and L^r-uniqueness. Our main result is that recurrence implies uniqueness of infinitesimally invariant measures, as well as existence and uniqueness of invariant measures. We can hence make in particular use of various explicit analytic criteria for recurrence that have been previously developed in the context of (generalized) Dirichlet forms and present diverse examples and counterexamples for uniqueness of infinitesimally invariant, as well as invariant measures and an example where L^1-uniqueness fails although pathwise uniqueness holds. Furthermore, we illustrate how our results can be applied to related work and vice versa. This is joint work with Haesung Lee.

Mixing time for a reaction-diffusion model

Kenkichi Tsunoda

In this talk we consider a superposition of a diffusively accelerated simple exclusion process with birth and death dynamics on a one-dimensional periodic domain. It is known that this particle system leads to a reaction-diffusion equation in a macroscopic scale. We discuss the mixing time of this model, which is a time needed to reach equilibrium with respect to the total-variation distance. In particular, we examine a phase transition in the inverse temperature. That is, when the inverse temperature is less than some critical temperature, the system exhibits fast mixing, otherwise, exponentially slow mixing. This talk is based on a joint work with R. Tanaka arXiv:2011.13158 and arXiv:2105.12965.

Regularity estimates for nonlocal operators related to nonsymmetric forms

Marvin Weidner

In this talk we consider parabolic equations driven by nonlocal operators with a nonsymmetric jumping kernel. Such operators consist of a fractional order diffusion part and a nonlocal drift part of lower order. Our emphasis is on Hölder regularity estimates and (full) Harnack inequalities for weak solutions to such equations. Moreover, we discuss two-sided estimates for the fundamental solution to the corresponding Cauchy problem. This talk is based on joint work with Moritz Kassmann.