

SPECTRAL THEORY IN MANY VARIABLES

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Let X and Y be vector spaces. It is shown that a mapping $f : X \rightarrow Y$ satisfies the functional equation

$$\begin{aligned}
 (†) \quad & mn \, {}_{mn-2}C_{k-2} f\left(\frac{x_1 + \cdots + x_{mn}}{mn}\right) + m \, {}_{mn-2}C_{k-1} \sum_{i=1}^n f\left(\frac{x_{mi-m+1} + \cdots + x_{mi}}{m}\right) \\
 & = k \sum_{1 \leq i_1 < \cdots < i_k \leq mn} f\left(\frac{x_{i_1} + \cdots + x_{i_k}}{k}\right)
 \end{aligned}$$

if and only if the mapping $f : X \rightarrow Y$ is additive, and prove the Cauchy–Rassias stability of the functional equation (†) in Banach modules over a unital C^* -algebra, and in Poisson Banach modules over a unital Poisson C^* -algebra. Let $\mathcal{A}$ and $\mathcal{B}$ be unital C^* -algebras, Poisson C^* -algebras, Poisson JC^* -algebras or Lie JC^* -algebras. As an application, we show that every almost homomorphism $h : \mathcal{A} \rightarrow \mathcal{B}$ of $\mathcal{A}$ into $\mathcal{B}$ is a homomorphism when $h(2^d u y) = h(2^d u)h(y)$ or $h(2^d u \circ y) = h(2^d u) \circ h(y)$ for all unitaries $u \in \mathcal{A}$, all $y \in \mathcal{A}$, and $d = 0, 1, 2, \dots$, and that every almost linear almost multiplicative mapping $h : \mathcal{A} \rightarrow \mathcal{B}$ is a homomorphism when $h(2x) = 2h(x)$ for all $x \in \mathcal{A}$.

Moreover, we prove the Cauchy–Rassias stability of homomorphisms in C^* -algebras, Poisson C^* -algebras, Poisson JC^* -algebras or Lie JC^* -algebras, and of Lie JC^* -algebra derivations in Lie JC^* -algebras.