

DYNAMICAL SYSTEMS IN GRAPH C^* -ALGEBRAS

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Let $C^*(E)$ be the universal C^* -algebra associated with a locally finite irreducible infinite graph E with the edge set E^1 . $C^*(E)$ is then generated by a family of partial isometries $\{s_\alpha\}$ indexed by the finite paths α in E satisfying certain relations. The words of generators of the form $s_\alpha s_\beta^*$ for a pair of paths α and β that range at a same vertex $r(\alpha) = r(\beta)$ and have same length generate an AF C^* -subalgebra $\mathcal{A}_E$. Also the C^* -algebra $\mathcal{D}_E$ generated by the projections $s_\alpha s_\alpha^*$ becomes a masa of $\mathcal{A}_E$ for which there exists a surjective $*$ -isomorphism $\rho : \mathcal{D}_E \rightarrow C_0(X_E)$, where X_E is the locally compact space of infinite paths in E . If $\phi_E : C^*(E) \rightarrow C^*(E)$ is the canonical cp map given by $\phi_E(x) = \sum_{e \in E^1} s_e x s_e^*$, its restriction to $\mathcal{D}_E$ is transformed to the shift map $\sigma_E : X_E \rightarrow X_E$ in the sense that $\rho(\phi_E(s_\alpha s_\alpha^*))(\beta) = \rho(s_\alpha s_\alpha^*)(\sigma_E(\beta))$, $\beta \in X_E$. It has been known that Voiculescu's topological entropy $ht(\phi_E|_{\mathcal{D}_E})$ coincides with the loop entropy $h_l(E)$ of E so that $ht(\phi_E|_{\mathcal{D}_E}) = h_{top}(\bar{\sigma}_E)$ follows, where $h_{top}(\bar{\sigma}_E)$ is the usual topological entropy of the continuous extension $\bar{\sigma}_E$ of σ_E to the one point compactification of X_E . For each vertex v of E we discuss dynamical systems $(\mathcal{A}_E(v), \phi_E)$ and provide an upper bound and a lower bound (the loop entropy of E) for $ht(\phi_E|_{\mathcal{A}_E(v)})$, where $\mathcal{A}_E(v)$ is the AF subalgebras of $\mathcal{A}_E$ generated by partial isometries $s_\alpha s_\beta^*$ such that $r(\alpha) = r(\beta) = v$.