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ON SOME OPERATOR UNITARILY EQUIVALENT TO A SCALAR MULTIPLE OF ITSELF

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Every weighted shift (unilateral or bilateral) T is unitarily equivalent to λT for all complex numbers λ with modulus one (see for instance [3]) and, as a consequence, its spectrum has circular symmetry about the origin. If a non-trivial T is unitarily equivalent to λT for some $|\lambda| \neq 1$, then T must be unbounded. Let q be a positive number with $q \neq 1$. Let T be a closed densely defined operator in a Hilbert space. If T satisfies

$$TT^* = q\,T^*T,$$

then T is called a *q-normal operator*. Elements obeying this relation in the algebraic sense appear at various places in the theory of quantum groups ([1] and the reference cited therein). It was proved ([2]) that a q-normal operator T has the property that T is unitarily equivalent to qT. This observation leads us to a natural question which operators are unitarily equivalent to a scalar multiple of themselves.

In this talk, I will report on the study of the class of closed densely defined operators T in a Hilbert space such that T is unitarily equivalent to qT. In particular, I will emphasize that such property in the case of weighted shifts is characterized in terms of a certain integer $k(S, q)$, which is uniquely determined by the weighted shift S and the spectrum of a weighted shift with such property coincides with the whole complex plane.

Reference

[1] A. Klimyk and K. Schmüdgen, *Quantum groups and their representations, Texts and Monographs in Physics*, Springer-Verlag New York-Heidelberg-Berlin, 1997.
[2] S. Ôta, Some classes of q-deformed operators, J. Operator Theory, **48**(2002), 151-186.
[3] A.L. Shields, Weighted shift operators and analytic function theory, Math. Surveys, American Mathematical Society **13**, Providence Rhode Island (1974)