

Structural and Spectral Properties of Hyponormal Pairs of Commuting Subnormal Operators

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The Lifting Problem for Commuting Subnormals (LPCS) asks for necessary and sufficient conditions for a pair of subnormal operators on Hilbert space to admit commuting normal extensions. It is well known that the commutativity of the pair is necessary but not sufficient, and it has recently been shown that the joint hyponormality of the pair is necessary but not sufficient. Moreover, while abstract solutions of LPCS exist, concrete solutions are only known in very specific situations. Our previous research (joint with J. Yoon and S.H. Lee) has shown that many of the nontrivial aspects of LPCS are best detected within the class \mathfrak{H}_1 of commuting hyponormal pairs of subnormal operators, so we focus our attention on this class. A large subclass of \mathfrak{H}_1 , the 2-variable weighted shifts with tensor core (denoted by \mathcal{TC}), has proved quite valuable in a related context. In the first part of this talk, I will present recent joint work with Sang Hoon Lee and Jasang Yoon, in which we provide a complete solution of LPCS within the class \mathcal{TC} .

In the second part of the talk, I will discuss the spectral properties of commuting pairs of subnormal operators. This is joint work with Jasang Yoon, and it deals with 2-variable weighted shifts. By contrast with all known results in the theory of (single and 2-variable) weighted shifts, we show that the Taylor spectrum can be disconnected. We do this by obtaining a simple sufficient condition that guarantees disconnectedness, based on the norms of the horizontal slices of the shift. We also show that for every $k \geq 1$ there exists a k -hyponormal 2-variable weighted shift whose horizontal and vertical slices have 1- or 2-atomic Berger measures, and whose Taylor spectrum is disconnected.

We use tools and techniques from multivariable operator theory, from our work on the Lifting Problem for Commuting Subnormals, and from the groupoid machinery developed by the author and P. Muhly to analyze the structure of C^* -algebras generated by multiplication operators on Reinhardt domains. As a by-product, we show that, for 2-variable weighted shifts, the Taylor essential spectrum is not necessarily the boundary of the Taylor spectrum.