Subscalarity of k-th Roots of Hyponormal Operators

B. P. Duggal and Ch. Benhida Northfield Avenue, 8 Redwood Grove, Ealing, London W5 4SZ, United Kingdom bpduggal@yahoo.co.uk

If a Hilbert space operator $A, A \in B(\mathcal{H})$, is a k-th root of a hyponormal or a p-hyponormal or an M-hyponormal operator such that the spectrum of A is contained in an angle $< 2\pi/k$ with vertex in the origin, then A is subscalar. In general, p-quasihyponormal operators fail to be subscalar; however, if a p-quasihyponormal operator $A \in B(\mathcal{H})$ satisfies $A^{-1}(0) \subseteq A^{*-1}(0)$, then A is subscalar. k-th roots of (p,q)-quasihyponormal operators satisfy Bishop's property (β) . Consequently, k-th roots A of p-hyponormal, M-hyponormal and (p,q)-quasihyponormal operators are not supercyclic, and if $\sigma(A)$ is thick then A has a nontrivial invariant subspace; furthermore, f(A+T) satisfies Weyl's theorem, and $f(A^*+T^*)$ satisfies a-Weyl's theorem, for every algebraic operator T which commutes with A and every f which is analytic in an open neighbourhood of $\sigma(A+T)$.