Abstracts of KOTAC
Volume 9(2007), 7–7

Reverse of a Log-majorization Inequality

Masatoshi Fujii
Department of Mathematics, Osaka Kyoiku University 582-8582, Japan
mfujii@cc.osaka-kyoiku.ac.jp

Bebiano-Lemos-Providência [1] showed the following log-majorization inequality:

For positive invertible operators A, B on a Hilbert space,

$$A^{\frac{1+t}{2}} B^t A^{\frac{1+t}{2}} \quad \prec_{(log)} \quad A^{\frac{1}{2}} (A^{\frac{s}{2}} B^s A^{\frac{s}{2}})^{\frac{t}{s}} A^{\frac{1}{2}}$$

holds for all $s \geq t \geq 0$.

In this talk, we propose a reverse inequality of it, precisely,

$$A^{\frac{1+t}{2}} B^t A^{\frac{1+t}{2}} \quad \succ_{(log)} \quad A^{\frac{1}{2}} (A^{\frac{s}{2}} B^s A^{\frac{s}{2}})^{\frac{t}{s}} A^{\frac{1}{2}}$$

holds for all $t \geq s \geq 1$.

Incidentally, their inequality follows from the Furuta inequality [2]F

If $A \geq B \geq 0$, then for each $r \geq 0$,

$$\left(A^{\frac{r}{2}} A^p A^{\frac{r}{2}}\right)^{\frac{1}{q}} \geq \left(A^{\frac{r}{2}} B^p A^{\frac{r}{2}}\right)^{\frac{1}{q}}$$

holds for $p \geq 0$ and $q \geq 1$ with $(1+r)q \geq p+r$.

Reference.

[1] N. Bebiano, R. Lemos and J. Providência, *Inequalities for quantum relative entropy*, Linear Algebra Appl., **401**(2005), 159–172.

[2] T. Furuta, $A \geq B \geq 0$ *assures* $(B^r A^p B^r)^{1/q} \geq B^{(p+2r)/q}$ *for* $r \geq 0, p \geq 0, q \geq 1$ *with* $(1+2r)q \geq p+2r$, Proc. Amer. Math. Soc., **101**(1987), 85–88.