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Cancellation for Inclusions of C*-algebras

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For two projections p, q in a C^*-algebra, we write $p \sim q$ if they are Murray-von Neumann equivalent. A C^*-algebra A is said to have *cancellation of projections* if whenever $p, q, r \in A$ are projections with $p \perp r$, $q \perp r$, and $p + r \sim q + r$, then $p \sim q$. If the matrix algebra $M_n(A)$ over A has cancellation of projections for each $n \in \mathbb{N}$, we simply say that A has *cancellation*. Every C^*-algebra with cancellation is stably finite. For a unital C^*-algebra A, if the topological stable rank $tsr(A)$ of A satisfies $tsr(A) = 1$, then A has cancellation. For a stably finite simple C^*-algebra A, it has been a long standing open question, settled negatively by A. S. Toms, whether cancellation implies $tsr(A) = 1$.

Let $1 \in A \subset B$ be a unital inclusion of C^*-algebras with index-finite type and with finite depth. In particular, B could be a crossed product $A_\alpha G$ of a unital C^*-algebra by a finite group. Our main result says that if A is simple, has topological stable rank 1, and satisfies Property (SP) (every hereditary C^*-subalgebra contains a nonzero projection), then B has cancellation. As an intermediate result, we show that the topological stable rank satisfies

$$tsr(B) \leq tsr(A) + n - 1.$$

if $1 \in A \subset B$ is an inclusion of C^*-algebras with common unit and $E\colon B \to A$ is a conditional expectation with index-finite type and a quasi-basis of n elements.