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Operators Admitting a Moment Sequence

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This paper is joint work with B. Chevreau, E. Ko, and C. Pearcy. In this paper $\mathcal{H}$ will always be a separable, infinite dimensional, complex Hilbert space, and $\mathcal{L}(\mathcal{H})$ will denote the algebra of all bounded linear operators on $\mathcal{H}$. As usual, $\mathbf{K} = \mathbf{K}(\mathcal{H})$ will denote the ideal of compact operators in $\mathcal{L}(\mathcal{H})$, and we write $\mathbb{N}\,[\mathbb{N}_0]$ for the set of positive integers. Following [1] and [3], we say that an operator T in $\mathcal{L}(\mathcal{H})$ *admits a moment sequence* if there exist nonzero vectors x and y in $\mathcal{H}$ and a (finite, regular) Borel measure μ supported on the spectrum $\sigma(T)$ of T such that $\langle T^n x, y\rangle = \int_{\sigma(T)} \lambda^n \, d\mu,\ n \in \mathbb{N}_0$.

In [1], Atzmon and Godefroy proved that if $\mathcal{X}$ is a real separable Banach space and T is a bounded linear operator on $\mathcal{X}$ that admits a moment sequence (with associated Borel measure μ supported on $\sigma(T) \subset \mathbb{R}$), then T has a nontrivial invariant subspace. Also, it is obvious that every T in $\mathcal{L}(\mathcal{H})$ that has a nontrivial invariant subspace (n.i.s.) admits a moment sequence, and Atzmon and Godefroy point in the direction of the possible equivalence of the two concepts. Thus one believes that the question of which operators in $\mathcal{L}(\mathcal{H})$ can be shown to have a moment sequence is worth further exploration.

Let $(\mathbf{N}+\mathbf{K})$ be the set of all operators T in $\mathcal{L}(\mathcal{H})$ that can be written as a sum $T = N + K$, where N is a normal operator and K is compact. We write, as usual, $\pi : \mathcal{L}(\mathcal{H}) \to \mathcal{L}(\mathcal{H})/\mathbf{K}$ for the Calkin map, and $\sigma_e(T) := \sigma(\pi(T))$, $\|T\|_e := \|\pi(T)\|$. In [3], they proved the following theorem

Theorem A. *Every $T \in (\mathbf{N}+\mathbf{K})$ admits a moment sequence.*

Theorem B ([2]). *Let $\mathcal{A}$ be a proper subalgebra of $\mathcal{L}(\mathcal{H})$ that is closed in the weak operator topology and contains the identity operator $1_{\mathcal{H}}$. Then there exist nonzero vectors x and y in $\mathcal{H}$ such that*

1) $\langle x, y\rangle \geq 0$, *and*

2) *the linear functional $\varphi \in \mathcal{A}^*$ defined by $\varphi(A) = \langle Ax, y\rangle$ satisfies $|\varphi(A)| \leq \langle x, y\rangle \|A\|_e$ for every $A \in \mathcal{A}$.*

In this paper, we discuss a new and simple proof of Theorem A by using Theorem B.

Corollary C. *Every T in $\mathcal{L}(\mathcal{H})$ that is either nonbiquasitriangular, essentially normal, or hyponormal admits a moment sequence.*

Theorem D. *Suppose $T \in \mathcal{L}(\mathcal{H})$ is almost hyponormal, and let X be any Hilbert-Schmidt operator in $\mathcal{L}(\mathcal{H})$. Then, if $T^*T - TT^* \notin C_1(\mathcal{H})$, the operator $T + X$ has a n.i.s.*

Theorem E. *Every operator in $\mathcal{L}(\mathcal{H})$ of the form $T + X$, where T is almost hyponormal and $X \in \mathcal{C}_2(\mathcal{H})$ admits a moment sequence.*

Theorem F. *Suppose $T \in \mathcal{L}(\mathcal{H})$ and $\sigma(T)$ contains at least one isolated point. Then T has a n.i.s. if and only if T admits a moment sequence.*

Proposition G. *Every $T = S + K \in \mathcal{L}(\mathcal{H})$ with S subnormal and $K \in \mathbf{K}$ has a moment sequence.*

References

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