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The Invariant Subspace Problem for Hyponormal Operators

Jaewoong Kim
Department of Mathematical Sciences, Seoul National University, Seoul, 151-747, Korea
kim2@snu.ac.kr

If $T \in L(H)$ then T is said to have a nontrivial invariant subspace if there is a subspace $\mathfrak{M}$ of H such that $\{0\} \neq \mathfrak{M} \neq H$ and $T\mathfrak{M} \subset \mathfrak{M}$. In this case we can represent T as

$$T = \begin{pmatrix} * & * \\ 0 & * \end{pmatrix} \quad \text{on } \mathfrak{M} \oplus \mathfrak{M}^{\perp}.$$

In 1932, J. von Neumann addressed the following problem:

If $\mathcal{X}$ is a Banach space of dim $\mathcal{X} \geq 2$, does $T \in L(\mathcal{X})$ have a nontrivial invariant subspace?

Today this is known as the *Invariant Subspace Problem*. In 1984, C.J. Read has shown that there exists an operator acting on ℓ_1 which has no nontrivial invariant subspace. However the invariant subspace problem remains still open for the cases of separable Hilbert spaces.

In this paper we show:

Theorem 1. Let $T \in L(H)$ be an operator such that $||p(T)|| \leq ||p||_{\sigma(T)}$ for every polynomial p. If $\operatorname{int} \widehat{\sigma(T)}$ is a union of C^2-Caratheodory domains then T has a nontrivial invariant subspace. (Here $\widehat{K}$ means the polynomially convex hull of K.)

Corollary 1. If $T \in L(H)$ is a hyponormal operator such that $\operatorname{int} \widehat{\sigma(T)}$ is a boundary of a C^2-Caratheodory domain then T has a nontrivial invariant subspace.