Abstracts of KOTAC
Volume 9(2007), 24–24

# Generalized Bebiano-Lemos-Providência Inequality

Masaru Tominaga
Toyama National College of Technology 13, Hongo-machi, Toyama-shi, 939-8630, Japan
mtommy@toyama-nct.ac.jp

In [1], Bebiano, Lemos and Providência gives a norm inequality (BLP norm inequality): For positive operators $A, B$ on Hilbert space

$$\|A^{\frac{1+t}{2}}B^tA^{\frac{1+t}{2}}\| \quad \le \quad \|A^{\frac{1}{2}}(A^{\frac{s}{2}}B^sA^{\frac{s}{2}})^{\frac{t}{s}}A^{\frac{1}{2}}\|$$

for all $s \ge t \ge 0$. We note that this BLP norm inequality is represented as the operator inequality

$$A\natural_{\frac{1}{p}}^{s}B^{p+s} \ \le \ A^{1+s} \text{ for some } p \ge 1 \text{ and } s \ge 0 \quad \Longrightarrow \quad B^{1+\frac{s}{p}} \ \le \ A^{1+\frac{s}{p}}.$$

In this talk, we discuss a precise inequality of this one. At a result, we have the following inequality:

$$A\natural_{\frac{1}{p}}^{s}B^{p+s} \ \le \ A^{1+s} \text{ for some } p \ge 1 \text{ and } s \ge 0 \quad \Longrightarrow \quad B^{1+s} \ \le \ A^{1+s}.$$

The Furuta inequality [2] plays an important role in our discussion:

$$A \ge B \ge 0 \quad \Longrightarrow \quad A^{1+r} \ \ge \ \left(A^{\frac{r}{2}}B^pA^{\frac{r}{2}}\right)^{\frac{1+r}{p+r}}$$

holds for $p \ge 1$ and $r \ge 0$.

Moreover we give the following norm inequality corresponding to Theorem F

$$\|A^{\frac{1+s}{2}}B^{1+s}A^{\frac{1+s}{2}}\|^{\frac{p+s}{p(1+s)}} \quad \le \quad \|A^{\frac{1}{2}}(A^{\frac{s}{2}}B^{p+s}A^{\frac{s}{2}})^{\frac{1}{p}}A^{\frac{1}{2}}\|$$

for all $p \ge 1$ and $s \ge 0$.

## References

[1] N. Bebiano, R. Lemos and J. Providência, *Inequalities for quantum relative entropy*, Linear Algebra Appl. **401**(2005), 159–172.

[2] T. Furuta, $A \ge B \ge 0$ *assures* $(B^rA^pB^r)^{1/q} \ge B^{(p+2r)/q}$ *for* $r \ge 0, p \ge 0, q \ge 1$ *with* $(1+2r)q \ge p+2r$, Proc. Amer. Math. Soc., **101**(1987), 85–88.