Abstracts of KOTAC
Volume 9(2007), 25–25

Branched Points and C*-algebras Associated with Complex Dynamical Systems

Yasuo Watatani
Department of Mathematical Sciences, Kyushu University, Hakozaki, Fukuoka, 812-8581, Japan
watatani@math.kyushu-u.ac.jp

This is a joint work with M. Izumi and T. Kajiwara.

In this talk we review some relations between complex dynamical systems of rational functions and associated C^*-algebras with the gauge action. In particular we study singularity structure (branched points) of the complex dynamical systems in terms of operator algebras.

For a branched covering, Deaconu and Muhly introduced a C^*-algebra associated with it using a r-discrete groupoid. We introduced a slightly different construction of a C^*-algebra $\mathcal{O}_R(\hat{\mathbb{C}})$ (resp. $\mathcal{O}_R(J_R)$ and $\mathcal{O}_R(F_R)$) associated with a rational function R on the Riemann sphere $\hat{\mathbb{C}}$ (resp. the Julia set J_R and the Fatou set F_R of R). The C^*-algebra $\mathcal{O}_R(\hat{\mathbb{C}})$ is the Cuntz-Pimsner algebra of a Hilbert bimodule over the C^*-algebra $C(\hat{\mathbb{C}})$ of the set of continuous functions and the other two are defined in a similar way.

We study the gauge action α on the C^*-algebra . If rational functions P and Q are topological conjugate, then the associated bimodules are isomorphic. Therefore the C^*-dynamical systems $(\mathcal{O}_P(\hat{\mathbb{C}}), \alpha)$ and $(\mathcal{O}_Q(\hat{\mathbb{C}}), \alpha)$ are conjugate. Hence the structure of the KMS states for the gauge action is an invariant for complex dynamical systems of rational functions up to topological conjugacy. We completely classify the KMS states for the gauge action of $\mathcal{O}_R(\hat{\mathbb{C}})$. If R has no exceptional points, then the gauge action has a phase transition at $\beta = \log \deg R$ in the following sense: In the region $0 \leq \beta < \log \deg R$, no KMS-state exists. A unique KMS-state exists at $\beta = \log \deg R$, which is of type $III_{1/\deg R}$ and corresponds to the Lyubich measure. The extreme β-KMS states at $\beta > \log \deg R$ are parameterized by the branched points of R and are factor states of type I. If R has exceptional points, then there appear additional β-KMS states for $0 < \beta \leq \log \deg R$ parameterized by exceptional points. We can recover the degree of R, the number of branched points, the number of exceptional points from the structure of the KMS states. The orbits of exceptional points are distinguished by 0-KMS states. Here we define a 0-KMS state to be an α-invariant tracial state.