Abstracts of KOTAC
Volume 12(2010), 2–3

A characterization of a coaction associated with the induced action

Hisashi Aoi

Department of Mathematical Sciences, Ritsumeikan University, Kusatsu, Shiga, 525-8577
Japan
aoi@fc.ritsumei.ac.jp

By [2], it is well-known that the continuous decompositions are obtained as the induced actions of the discrete decomposition for III_λ factor $(0 < \lambda < 1)$. This result can be phrased in terms of group coactions by using the Fourier transformation.

In this talk, we generalized the result to (not necessarily abelian) locally compact group coaction with its dual action.

For this, we recall that the coaction α of a locally compact group K on a von Neumann algebra A is a unital normal $*$-isomorphism from A into $W^*(K) \otimes A$ which satisfies the following:

$$(\mathrm{id} \otimes \alpha) \circ \alpha = (\Delta_K \otimes \mathrm{id}) \circ \alpha,$$

where $W^*(K)$ is the von Neumann algebra which is generated by the left regular representation of K with the coproduct $\Delta_K : W^*(K) \ni \lambda_K(k) \mapsto \lambda_K(k) \otimes \lambda_K(k) \in W^*(K) \otimes W^*(K)$. (We note that, by using the Fourier transformation, if K is abelian, then coactions of K coincide with actions of the dual group $\widehat{K}$.)

We also recall that, for each action β of H on a von Neumann algebra P, we define (mutually commute) actions γ of H and κ of K on $L^\infty(K) \otimes P$ by the following:

$$\gamma_h(X)(l) := \beta_h(X(lh)),\ \kappa_k(X(l)) := X(k^{-1}l) \quad (X \in L^\infty(K) \otimes P,\ h \in H,\ k, l \in K).$$

The restriction of κ to $Q := (L^\infty(K) \otimes P)^\gamma$. is called the induced action by β.

We will prove the following:

Theorem 1 ([1], cf. [2]) *Let α be a coaction of a locally compact group K on a properly infinite von Neumann algebra A, and H be a closed subgroup of K with the $*$-isomorphism $I : W^*(H) \ni \lambda_H(h) \mapsto \lambda_K(h) \in W^*(K)$. Then the following are equivalent:*

1. *There exists an α-1-cocycle R such that the subalgebra $\{(\mathrm{id} \otimes \omega)_R \alpha(a) : a \in A,\ \omega \in A_*\}''$ is contained in $I(W^*(H))$.*

2. *The dual action $\widehat{\alpha}$ of K on $\widehat{K}\ {}_\alpha\!\ltimes A$ is induced by some action of H.*

3. *There exists an injective $*$-homomorphism Θ from $L^\infty(K/H)$ to the center of $\widehat{K}\ {}_\alpha\!\ltimes A$ such that $\Theta \circ \ell_k = \widehat{\alpha}_k \circ \Theta$ for each $k \in K$, where ℓ_k comes from the left translation by k on K/H.*

Moreover, if one of the above conditions occurs, then there exists a coaction α' of H on A such that the system $\{\widehat{K}\ {}_\alpha\!\ltimes A, \widehat{\alpha}\}$ is induced by the system $\{\widehat{H}\ {}_{\alpha'}\!\ltimes A, \widehat{\alpha'}\}$.

References

[1] H. Aoi, *A characterization of a coaction reduced to that of a closed subgroup*, Tokyo Journal of Math., **30** (2008), no.2, 311–324

[2] M. Takesaki, *Duality for crossed products and the structure of von Neumann algebras of type III*, Acta Math., **131** (1973), 249–310.