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A reverse Ando inequality on positive linear maps

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Pólya-Szegö showed the following reverse of Cauchy inequality: If the real numbers a_i and b_i $(i = 1, 2, \cdots, n)$ satisfy the conditions $0 < m_1 \le a_i \le M_1$ and $0 < m_2 \le b_i \le M_2$, then

$$1 \le \frac{\sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2}{\left(\sum_{i=1}^{n} a_i b_i\right)^2} \le \frac{(M_1M_2 + m_1m_2)^2}{4m_1m_2M_1M_2}.$$

Non-commutative versions of the Pólya-Szegö inequality has been studied by many mathematicians. For example, Ando considered Cauchy inequality for the Hadamard product:

$$A \circ B \le (A^2 \circ I)^{\frac{1}{2}} (B^2 \circ I)^{\frac{1}{2}}.$$

By using a reverse Ando inequlity on positive linear maps, we have Pólya-Szegö inequality for the Hadamard product: If A and B are positive operators on a Hilber space H such that $0 < mI \le A, B \le MI$ for some scalars $0 < m \le M$, then

$$(A^2 \circ I)^{\frac{1}{2}} (B^2 \circ I)^{\frac{1}{2}} \le \frac{M^2 + m^2}{2Mm} A \circ B.$$

In this talk, we show n-variable Pólya-Szegö inequality for the Hadamard product by using the Specht ratio.

References

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