

Completely positive maps on Coxeter groups with  
 applications to noncommutative von Neumann inequality,  
 deformed Fock spaces and BMV conjecture

Marek Boejko

University of Wroclaw, Poland

1. Quasi-multiplicative operator-valued functions  $P$  on permutation(Coxeter)groups and free groups  $G$  with respect natural length functions  $L(x)$  = minimal numbers of generators in the word  $x \in G$ , i.e.

$P(xy) = P(x)P(y)$ , if  $L(xy) = L(x) + L(y)$ , for  $x, y$  in group  $G$ ,

and

$$P(x^{-1}) = P(x)^* \text{ and } P(e) = I.$$

2. Completely positive maps of quasi-multiplicative functions on a group  $C^*$ –algebras of permutations and free groups.

3. Applications to noncommutative von Neumann inequality:

For arbitrary contractions  $T_j$  on a Hilbert space and arbitrary non-commutative polynomial on  $n$  variables we have

$$\|p(T_1, \dots, T_n)\| < \sup\|p(U_1, \dots, U_n)\|$$

where  $U_j$  are finite dimensional unitary matrices.

Case  $n = 1$  is the classical von Neumann inequality.

4. For arbitrary self-adjoint contraction  $T$  on tensor product of Hilbert spaces,  $H \otimes H$ , we give construction of large class of deformed Fock spaces and many examples of von Neumann algebras which are factors i.e. centrum is trivial.

5. We prove for large class of self-adjoint operators  $A$  and  $B$  that the Bessis-Moussa-Villani(BMV) conjecture is true i.e.:

The function on the real line  $F(x) = \text{tr}(\exp(A + ixB))$  is positive definite.

6. Normal law and  $q$ -Gaussian laws for  $q \in [0, 1]$ , are free infinitely divisible.

References

- [1]. M.Anshelevich,S.Belinschi,M.Bozejko and F.Lehner, Free infinite divisibility for  $q$ -Gaussian,Preprint,arXiv 2010.
- [2]. S.Belinschi,M.Bozejko,F.Lehner and R.Speicher, The normal law is free infinitely divisible,Preprint ,arXiv 2009.
- [3]. M.Boejko, Positive definite kernels,lenth functions on groups and a non-commutative von Neumann inequality, Studia Math.95(1989),107-118.
- [4]. M.Bozejko. Bessis-Moussa- Vilani conjecture and generalized Gaussian random variables, Inf.Dim.Analysis,Quantum Prob and Related Topics,11(2008),313-321.
- [5]. M.Bozejko.R.Speicher, Completely positive maps on Coxeter groups,deformed commutation relations and operator spaces, Math Ann. 300(1994),97-120.
- [6]. G.Pisier,Similarity Problems and Completely Bounded Maps, Lecture Notes in Math.1618,2001.
- [7].G.Pisier,Introduction to Operator Space Theory,2003.