

Completely positive maps on Coxeter groups with applications to noncommutative von Neumann inequality, deformed Fock spaces and BMV conjecture

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1. Quasi-multiplicative operator-valued functions P on permutation(Coxeter)groups and free groups G with respect natural length functions $L(x)$ = minimal numbers of generators in the word $x \in G$, i.e.

$$P(xy) = P(x)P(y), \text{ if } L(xy) = L(x) + L(y), \text{ for } x, y \text{ in a group } G,$$

and

$$P(x^{-1}) = P(x)^* \text{ and } P(e) = I.$$

2. Completely positive maps of quasi-multiplicative functions on a group C^* –algebras of permutations and free groups.

3. Applications to noncommutative von Neumann inequality:

For arbitrary contractions T_j on a Hilbert space and arbitrary non-commutative polynomial on n variables we have

$$\|p(T_1, \dots, T_n)\| < \sup \|p(U_1, \dots, U_n)\|$$

where U_j are finite dimensional unitary matrices.

Case $n = 1$ is the classical von Neumann inequality.

4. For arbitrary self-adjoint contraction T on tensor product of Hilbert spaces, $H \otimes H$, we give construction of large class of deformed Fock spaces and many examples of von Neumann algebras which are factors i.e. centrum is trivial.

5. We prove for large class of self-adjoint operators A and B that the Bessis-Moussa-Villani(BMV) conjecture is true i.e.:

The function on the real line $F(x) = \text{tr}(\exp(A + ixB))$ is positive definite.

6. Normal law and q -Gaussian laws for $q \in [0, 1]$, are free infinitely divisible.

References

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