

## Unitary Operators and Pairs of Subalgebras

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A unitary operator  $u$  in a  $C^*$ -algebra  $M$  implements the  $*$ -automorphism  $Adu : Ad u(x) = uxu^*$  for all  $x$  in  $M$ . For a given subalgebra  $A$  of  $M$ , the inclusion  $A \subset M$  is isomorphic to the inclusion  $\theta(A) \subset M$  for all automorphism  $\theta$  of  $M$ . However, relations between two subalgebras  $A$  and  $uAu^*$  depends on the properties of  $u$  connected to  $A$ . In this talk, we show some results for unitary operators in connection with subalgebras. First, as one of the most elementary example, we pick up unitary matrices and maximal abelian  $*$ -subalgebras of the  $n \times n$  complex matrices  $M_n(\mathbb{C})$ . We discuss them in a relation to states of  $M_n(\mathbb{C})$ . Next, we extend our discussion on maximal abelian  $*$ -subalgebras of  $M_n(\mathbb{C})$  to subfactors  $N$  of finite factors  $M$ , and show relations between  $N$  and unitary operators of  $M$ .