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Partially normal composition operators via directed trees

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A pair (V, E) is a *directed graph* if V is a nonempty and E is a subset of $V \times V \backslash \{(v, v) : v \in V\}$. An element of V is called a *node* of $\mathcal{G}$. Set $\mathrm{Chi}(u) = \{v \in V : (u, v) \in E\}$, $u \in V$. For $u \in V$, the cardinality of $\mathrm{Chi}(u)$ called to be the *degree* of u. Put $\mathrm{Gen}^k(u) = \cup_{v \in \mathrm{Gen}^{k-1}(u)} \mathrm{Chi}(v)$, $u \in V$, where $\mathrm{Gen}^0(u) = \{u\}$. The set $\mathrm{Gen}^k(u)$ is called *generation* k of u. A graph $\mathcal{G} = (V, E, \mu)$ is a *weighted directed graph* if (V, E) be a directed graph and $(V, \mathcal{P}(V), \mu)$ be a σ-finite measure space on V, where $\mathcal{P}(V)$ is the power set of V. For a node $u \in V$, we write m_u for the point mass $\mu(u)$. If, for a given node $u \in V$, there exists a unique node $v \in V$ such that $(v, u) \in E$, then we say that u has a *parent* v and write $\mathrm{par}(u)$ for v. A node v of $\mathcal{G}$ is called a *root* of $\mathcal{G}$ if there is no node u of $\mathcal{G}$ such that (u, v) is an edge of $\mathcal{G}$. A graph $\mathcal{G} = (V, E, \mu)$ is a *weighted directed tree* if $\mathcal{G}$ is a weighted directed graph such that $\mathcal{G}$ is connected, $\mathcal{G}$ has no circuits, and each node $v \in V \backslash \{\mathrm{root}\}$ has a parent. Given a graph $\mathcal{G}$ and a sequence of graphs $\mathcal{G}_n$, we say that $\mathcal{G}$ is *an equivalent-limit* of $\{\mathcal{G}_n\}$, and write $e\text{-}\lim \mathcal{G}_n = \mathcal{G}$ if there exist graphs $H_1,...,H_n,...$ such that $\mathcal{G}_i \backsimeq H_i$, $i = 1, 2, ...$, and $\lim_{n\to\infty} H_n = \mathcal{G}$. Let $\mathcal{G}$ be weighted directed trees and let T be the measurable transformation V corresponded by $\mathcal{G}$. The directed tree $\mathcal{G}$ is *p-hyponormal* if the composition operator C_T relevant to $\mathcal{G}$ is p-hyponormal. We call a (sub)directed tree consisting of one node not a root, its children and their children (including the point mass data) *a system* $\mathcal{S}$. We say that, for any node n in a directed graph not a root, the subgraph system in which n falls in generation 1 is *a neighborhood* of n and may write $\mathcal{S}(n)$. Let $D_{\mathcal{G}}$ be the maximum of the degrees and $M_{\mathcal{G}} := \sup_{u \in V} \left\{ \sum_{v \in \mathrm{Chi}(u)} m_v / m_u \right\}$,

$$\sum_{u \in \mathrm{Chi}(\mathrm{root})} \left(\frac{m_u}{\mu(\mathrm{Chi}(u))} \right)^p \cdot m_u \le \left(\frac{m_{\mathrm{root}}}{\mu(\{\mathrm{root}\} \cup \mathrm{Chi}(\mathrm{root}))} \right)^p \cdot \mu(\mathrm{Chi}(\mathrm{root})). \quad (\dagger)$$

We introduce one of our results as following.

Theorem. *Let $M \in \mathbb{R}_+$, $D \in \mathbb{N}$, $D \ge 2$, $p \in \mathbb{R}_+$ and $\epsilon > 0$ be arbitrary. Then there exists a p-hyponormal directed tree $\mathcal{E} = \mathcal{E}(M, D, \epsilon, p)$ with $D_{\mathcal{E}} = D$ and $M_{\mathcal{E}} < M + \epsilon$, and such that for all trees $\mathcal{G}$ with $D_{\mathcal{G}} \le D$ and $M_{\mathcal{G}} \le M$, the following assertions hold.*

(i) *If $\mathcal{G}$ has no root, $\mathcal{G}$ is p-hyponormal if and only if for every node n of $\mathcal{G}$, $\mathcal{S}(n) = e\text{-}\lim_{j\to\infty} \mathcal{S}_j$ with $\mathcal{S}_j$ a system in $\mathcal{E}$ for all j.*

(ii) *If $\mathcal{G}$ has a root, then the test (†) for p-hyponormal of the root r holds and $\mathcal{G}$ has the property in* (i).

In particular, if $D = 1$, then we may obtain the analogous result with $D_{\mathcal{E}} = 2$. Also, $\mathcal{E}$ has the additional property that $m_v \in \mathbb{Q}_+$ for all $v \in \mathcal{E}$.