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On rank-one perturbations of diagonal operators

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Let $\mathcal{H}$ be a separable, infinite dimensional, complex Hilbert space, and denote by $\mathcal{L}(\mathcal{H})$ the algebra of all bounded linear operators on $\mathcal{H}$. If $\Lambda = \{\lambda_n\}_{n\in\mathbb{N}}$ is any bounded sequence in $\mathbb{C}$, we write D_Λ for the normal operator in $\mathcal{L}(\mathcal{H})$ determined by the equations $D_\Lambda(e_n) = \lambda_n e_n,\ n \in \mathbb{N}$. An operator T in $\mathcal{L}(\mathcal{H})$ is called a rank-one perturbation of a diagonalizable normal operator if there exist nonzero vectors $u = \sum_{n\in\mathbb{N}} \alpha_n e_n$ and $v = \sum_{n\in\mathbb{N}} \beta_n e_n$ in $\mathcal{H}$ and a bounded sequence $\Lambda = \{\lambda_n\}_{n\in\mathbb{N}}$ in $\mathbb{C}$ such that T is unitarily equivalent to the operator $D_\Lambda + u \otimes v$, where, as usual, $u \otimes v$ is the operator of rank one defined by $(u \otimes v)(x) = \langle x, v\rangle u,\ x \in \mathcal{H}$. The notation $\{\alpha_n\}_{n\in\mathbb{N}}$ and $\{\beta_n\}_{n\in\mathbb{N}}$ for the Fourier coefficients of u and v, respectively, relative to the fixed basis $\{e_n\}$. In this talk, we discuss several properties such as invariant subspaces, commutants, point spectrums, inequalities, and quasisimilarity questions about rank-one perturbations of a diagonalizable normal operator. (This is joint work with Ciprian Foias, Eungil Ko, and Carl Pearcy.)