

When is hyponormality for 2-variable weighted shifts invariant under powers?

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Joint work with Raul Curto

For 2-variable weighted shifts $W_{(\alpha,\beta)} \equiv (T_1, T_2)$ we study the invariance of (joint) k -hyponormality under the action

$$(h, \ell) \mapsto W_{(\alpha,\beta)}^{(h,\ell)} := (T_1^h, T_2^\ell) \quad (h, \ell \geq 1).$$

We show that for every $k \geq 1$ there exists $W_{(\alpha,\beta)}$ such that $W_{(\alpha,\beta)}^{(h,\ell)}$ is k -hyponormal (all $h \geq 2, \ell \geq 1$) but $W_{(\alpha,\beta)}$ is not k -hyponormal.

On the positive side, for a class of 2-variable weighted shifts with tensor core we find a computable necessary condition for invariance.

Next, we exhibit a large nontrivial class for which hyponormality is indeed invariant under *all* powers; moreover, for this class 2-hyponormality automatically implies subnormality.

Finally, we show that there exists a 2-hyponormal $W_{(\alpha,\beta)}$ such that $W_{(\alpha,\beta)}^{(2,1)}$ is not 2-hyponormal.

Our results partially depend on new formulas for the determinant of generalized Hilbert matrices and on criteria for their positive semi-definiteness.