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ABSTRACTS

Operator Theory and Its Applications

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The Riemannian Mean of Positive Matrices

Rajendra Bhatia
Indian Statistical Institute, New Delhi, India

A longstanding problem of defining a geometric mean of n positive definite matrices with right operator theoretic properties was solved in 2004. One of the candidates, the Riemannian barycentre, has long been studied by geometers. The demands of operator theory have led to a new understanding of this classical geometric object. Operator monotonocity of this mean was proved in 2010, and different proofs found since then provide both interesting techniques and new insights. We will describe some of them in this talk.

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Structural and spectral properties of (jointly) hyponormal 2-variable weighted shifts

Raúl E. Curto
University of Iowa, USA
raul-curto@uiowa.edu

(This talk is based on joint work with Jasang Yoon and Sang Hoon Lee.)

We study structural and spectral properties of (jointly) hyponormal 2-variable weighted shifts with commuting subnormal components. By contrast with all known results in the theory of (single and 2-variable) weighted shifts, we show that the Taylor spectrum of such commuting pairs can be *disconnected*. We do this by obtaining a simple sufficient condition that guarantees disconnectedness, based on the norms of the horizontal slices of the shift. We also show that for every $k \geq 1$ there exists a k-hyponormal 2-variable weighted shift whose horizontal and vertical slices have 1- or 2-atomic Berger measures, and whose Taylor spectrum is disconnected.

We use tools and techniques from multivariable operator theory, from our previous work on the Lifting Problem for Commuting Subnormals and on k-hyponormality of 2-variable weighted shifts, and from the groupoid machinery developed by the author and P. Muhly to analyze the structure of C^*-algebras generated by multiplication operators on Reinhardt domains. As a by-product, we show that, for 2-variable weighted shifts, the Taylor essential spectrum is not necessarily the boundary of the Taylor spectrum.

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Proper lattices and spectrally invariant subspaces

Slaviša V. Djordjević
BUAP, Río Verde y Av. San Claudio, San Manuel, Puebla, Pue. 72570, Mexico
slavdj@fcfm.buap.mx

A proper lattices of X is a pair (A, L) composed by a bounded linear operator A on X and its invariant finite-dimensional subspace L. The set of all proper lattices of X we denote $Pl(X)$. For $(A, L) \in Pl(X)$, the operator A induces two operators, the restriction operator $A_{|L}$ and the operator $\widehat{A_L}$ from the quotient X/L into itself, i.e. $\widehat{A_L}(\pi(y)) = \pi(A(y))$, where π is the natural homoeomorphism between X and the quotient space X/L.

In this note its shown that (A, L) is a proper lattices if and only if there are the finite sequence of eigenvalues $\{\lambda_1, \ldots, \lambda_n\} \in \sigma_p(A)$ and the appropriate set of linear independent eigenvectors $\{x_1, \ldots, x_n\}$ such that $L = \mathcal{L}(x_1, \ldots, x_n)$. Moreover, λ_i is a simple pole of A if and only if $\lambda_i \notin \sigma(\widehat{A_L})$.

Follow this concept we can define spectrally invariant (finite dimensional) subspaces of linear operator T like invariant subspace E such that $\sigma(T_{|E}) \cap \sigma(\widehat{T_E}) = \emptyset$. Also, we gave it some properties of stability of spectrally invariant subspaces.

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On simple permanence

Robin E. Harte
Trinity College, Ireland
rhartere@gmail.com

"Simple permanence" is one of several variants of "spectral permanence", which are curiously inter related.

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An order-like relation induced by the Jensen inequality

Tomohiro Hayashi
Nagoya Institute of Technology, Gokiso-cho, Showa-ku, Nagoya, Aichi, 466-8555, Japan
hayashi.tomohiro@nitech.ac.jp

Let $\mathfrak{H}$ be a Hilbert space with an inner product $\langle \cdot, \cdot \rangle$ and we denote by $B(\mathfrak{H})$ the set of all bounded linear operators on $\mathfrak{H}$. If two positive operators $A, B \in B(\mathfrak{H})$ satisfy

$$\langle A^{\frac{1}{2}}\xi, \xi \rangle \leq \langle B\xi, \xi \rangle^{\frac{1}{2}}$$

for any unit vector $\xi \in \mathfrak{H}$, we write

$$A \trianglelefteq B.$$

The usual order $A \leq B$ implies that $A \trianglelefteq B$. This is a simple consequence of the Jensen inequality as follows.

$$\langle A^{\frac{1}{2}}\xi, \xi \rangle \leq \langle A\xi, \xi \rangle^{\frac{1}{2}} \leq \langle B\xi, \xi \rangle^{\frac{1}{2}}.$$

The following proposition is a characterization of this relation.

Proposition 1 *For two positive operators $A, B \in B(\mathfrak{H})$, the following conditions are equivalent.*

1. $A^2 \trianglelefteq B^2$.
2. $A \leq \frac{1}{2}(\frac{1}{t}B^2 + t)$ *for any positive number* t.
3. *There exists a contraction C satisfying* $CB + BC^* = 2A$

Here we remark that by the arithmetic-geometric mean inequality we have

$$B \leq \frac{1}{2}\Big(\frac{1}{t}B^2 + t\Big).$$

We might expect that the relation "$\trianglelefteq$" defines a new order. However there exit positive matrices A, B and C such that both $A \trianglelefteq B$ and $B \trianglelefteq C$ hold while $A \trianglelefteq C$ does not hold. The main result in this talk is the following.

Theorem 2 *Let $A, B \in B(\mathfrak{H})$ be two positive invertible operators. If they satisfy $A \trianglelefteq B$ and $B \trianglelefteq A$, then we have $A = B$.*

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Analytic functional calculus of operator convex/monotone functions

Fumio Hiai
Tohoku University, Japan
fumio.hiai@gmail.com

We consider a real function f on an open interval (a, b), $-\infty \leq a < b \leq \infty$, which is assumed to be continuously extended to an analytic function in the upper half-plane $\mathbb{C}^+$. By reflection principle, f is further extended to the lower half-plane $\mathbb{C}^-$ in such a way that f is analytic in $(\mathbb{C} \setminus \mathbb{R}) \cup (a, b)$. Let $\mathcal{H}$ be a (separable) Hilbert space and $B(\mathcal{H})$ be the set of all bounded operators on $\mathcal{H}$. For $X \in B(\mathcal{H})$ let $\operatorname{Re} X$ and $\operatorname{Im} X$ denote the real and the imaginary parts of X, respectively. For a self-adjoint $A \in B(\mathcal{H})$ we write $A > 0$ to mean that A is positive and invertible. For every $X \in B(\mathcal{H})$ with the spectrum $\sigma(X) \subset (\mathbb{C} \setminus \mathbb{R}) \cup (a, b)$ one can define the analytic functional calculus

$$f(X) := \frac{1}{2\pi i} \int_\Gamma f(\zeta)(\zeta I - X)^{-1} \, d\zeta,$$

where Γ is a piecewise smooth closed curve in $(\mathbb{C} \setminus \mathbb{R}) \cup (a, b)$ surrounding $\sigma(X)$. In particular, if $\operatorname{Im} X > 0$ then $\sigma(X) \subset \mathbb{C}^+$, and if $a < \operatorname{Re} X < b$ then $\sigma(X) \subset \{z \in \mathbb{C} : a < \operatorname{Re} z < b\}$, so the functional calculus $f(X)$ can be defined in these cases.

We characterize the operator convexity/monotonicity of f by inequalities between $\operatorname{Re} f(X)$ and $f(\operatorname{Re} X)$ and by positivity of $\operatorname{Im} f(X)$ for the analytic functional calculus $f(X)$. For instance, we show

- f is operator convex if and only if $\operatorname{Re} f(X) \leq f(\operatorname{Re} X)$ for every $X \in B(\mathcal{H})$ with $a < \operatorname{Re} X < b$.
- f is operator monotone if and only if $\operatorname{Im} f(X) \geq 0$ for every $X \in B(\mathcal{H})$ with $\operatorname{Im} X > 0$.

The latter result is regarded as the operator version of Löwner's characterization of operator monotone functions via analytic continuation to the upper half-plane. Furthermore, we show characterizations of the operator k-tonicity (the higher order operator monotonicity introduced by Franz, Hiai and Ricard) for functions on (a, b).

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On the generalized Powers-Størmer inequality

Dinh Trung Hoa
Duy Tan University, K7/25 Quang Trung, Danang, Vietnam
dinhtrunghoa@duytan.edu.vn

(This talk is based on joint work with Professor H.Osaka and Ho Minh Toan)

A generalization of Powers-Størmer's inequality for operator monotone functions on $(0, +\infty)$ and for positive normal linear functional on general C^*-algebras will be proved. It also will be shown that considered inequality characterizes the tracial functionals on C^*-algebras.

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Unbounded subnormal weighted shifts on directed trees

Il Bong Jung
Kyungpook National University, Daegu 702-701, Korea
ibjung@knu.ac.kr

A new method of verifying the subnormality of densely defined operators on Hilbert space based on an approximation technique is proposed. Diverse sufficient conditions for subnormality of unbounded weighted shifts on directed trees are established. An approach to this issue via consistent systems of probability measures is invented. The role played by determinate Stieltjes moment sequences is elucidated. Lamberts characterization of subnormality of bounded operators is shown to be valid for unbounded weighted shifts on directed trees that have sufficiently many quasi-analytic vectors, which is a new phenomenon in this area.

(This is a joint work with P. Budzynski, Z. Jablonski and J. Stochel.)

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Normal block Toeplitz operators

Dong-O Kang
Seoul National University, Seoul, Korea
skylover@snu.ac.kr

In [GuZ], the normality of block Toeplitz operators was characterized. This characterization is quite complicated, in comparison with the characterization of the normality of Toeplitz operators with scalar-valued symbols by A. Brown and P. Halmos. After several years, a simple characterization of normality of a block Toeplitz operator T_Φ was given in [GHR] when the determinant of the coanalytic part of Φ is a nonzero function. In this note, we will show that the same characterization is still valid for block Toeplitz operators with general matrix-valued symbols.

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[GHR] C. Gu, J. Hendricks and D. Rutherford, *Hyponormality of block Toeplitz operators*, Pacific J. Math. **223** (2006), 95–111.

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Crossed products of graph C^*-algebras by compact quantum group coactions

Dong Woon Kim
Seoul National University, Seoul, Korea
redpgr@dreamwiz.com

It is known that labeling the edges of a directed graph E with elements of a discrete group G induces a coaction of G on the graph C^*-algebra $C^*(E)$ and moreover the crossed product is isomorphic to the graph C^*-algebra associated to the skew-product graph corresponding to the labeling. Motivated by the fact that a group C^*-algebra $C^*(G)$ of a discrete group G is a compact quantum group, we consider labeling in the context of compact quantum groups and show that labeling the edges of a directed graph with one dimensional unitary corepresentations of a compact quantum group A induces a coaction of A on the graph C^*-algebra as in the case of labeling with group elements and a similar isomorphism result holds.

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Linear operators in the class θ and their properties

Jaewoong Kim
Seoul National University, Seoul, Korea
kim2@snu.ac.kr

In this talk, we introduce linear operators in the class θ, which is the set of operators T such that T^*T and $T + T^*$ commute. First we talk about spectral properties of operators in the class θ. By using these properties, we show that some operators in the class θ have a nontrivial invariant subspace. We also give a family of operators in the class θ which are reductive, i.e., their invariant subspaces are reducing

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The Resolvent average on symmetric cones

Sang Ho Kum
Chungbuk National University, Cheongju, Korea

Recently Bauschke et al. introduced a very interesting and new notion of proximal average in the context of convex analysis, and studied this subject systemically in [3–7] from various viewpoints. In addition, this new concept was applied to positive semidefinite matrices under the name of resolvent average, and basic properties of the resolvent average are successfully established by themselves from a totally different view and techniques of convex analysis rather than the classical matrix analysis. Inspired by their works and the well-known fact that the convex cone of positive definite matrices is a typical example of a symmetric cone, we study the resolvent average on symmetric cones, and derive corresponding results in a different manner based on a purely Jordan algebraic technique.

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The Ghost of a Departed Quantity: The Infinite-Dimensional Karcher Mean

Jimmie Lawson
Louisiana State University, Baton Rouge, LA 70803, USA

The least-squares mean on Ω, the set of positive definite $n \times n$-matrices, assigns to a k-tuple $(A_1, \dots, A_k) \in \Omega^k$ the (unique) matrix $\Lambda(A_1, \dots, A_k) \in \Omega$ that minimizes $F(X) = \sum_{i=1}^{k} d^2(X, A_i)$, where d is the standard trace metric on Ω. The least-squares mean has an alternative characterization as the point at which the gradient ∇F vanishes, i.e., as the unique solution of the Karcher equation $\sum_{i=1}^{n} \log(X^{-1/2} A_i X^{-1/2}) = 0$, and for this reason is frequently referred to as the Karcher mean. This mean easily extends to the weighted setting, has been shown to have a very nice theory, and has recently become an important tool for the averaging and study of positive definite matrices.

In recent work Yongdo Lim and the presenter have shown that this mean extends, in its general weighted form, to the infinite-dimensional setting of positive operators on a Hilbert space (more generally the positive elements of a C^*-algebra) and retains most of its attractive properties. However, in the infinite-dimensional setting there is no analog to the trace metric, so no least-squares characterization, only the alternative characterization as the vanishing of the Karcher equation, the "ghostly" remains of the gradient. We also introduce power means P_t in the infinite-dimensional setting and show that the Karcher mean is the strong limit of the monotonically decreasing family of power means as $t \to 0^+$. A third important characterization is the realization of the Karcher mean as the strong limit of least-square means defined on positive operators from the Hilbert-Schmidt extended algebra. We show each of these characterizations provide important insights about the mean.

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Distance k-graph of hypercubes and Hermite polynomial

Hun Hee Lee
Chungbuk National University, Cheongju, Korea
hhlee@chungbuk.ac.kr

We will use quantum probabilistic approach to analyze asymptotic behavior of the distance k-graph of hypercubes. We will briefly review basics of quantum probability and explain that Hermite polynomial of the standard normal variable can be obtained as a scaled limit of the distance k-graph of hypercubes.

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Quantum Information Science and Preserver problems

Chi-Kwong Li
College of William and Mary; University of Hong Kong
math.ckli@gmail.com

Preserver problems concern the characterization of mappings on matrices or operators with some special properties. This talk will focus on recent results on preserver problems arising in quantum information science including the preservers of decomposable states, and different functions on tensor product spaces. Open problems and current work will be mentioned.

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Quantum entanglement and positive linear maps

Hiroyuki Osaka
Department of Mathematical Sciences, Ritsumeikan University, Kusatsu, Shiga, 525-8577 Japan
osaka@se.ritsumei.ac.jp

Let M_n be the C*-algebra of all $n \times n$ matrices over the complex field $\mathbb{C}$ and $P[M_m, M_n]$ denote the set of all positive linear maps from M_m to M_n. A positive semi-definite matrix $A \in (M_m \otimes M_n)^+$ is said be entangled if $A \in (M_m \otimes M_n)^+ \setminus (M_m^+ \otimes M_n^+)$. Then using the duality between the space $M_m \otimes M_n$ and the space $L(M_m, M_n)$ by $\langle A, \phi \rangle = \mathrm{Tr}(AC_\phi^t) = \mathrm{Tr}(C_\phi A^t)$ $(C_\phi = (\phi(E_{ij}))$ we have $\phi \in P[M_m, M_n]$ if and only if $\langle A, \phi \rangle \geq 0$ for each $A \in M_m^+ \otimes M_n^+$. If we consider the cone of $\mathbb{T} = \{A \in (M_m \otimes M_n)^+ \mid A^\tau \in (M_m \otimes M_n)^+\}$ (τ is called a partial transpose such that $(x \otimes y)^\tau = x^t \otimes y$ for $x \otimes y \in M_m \otimes M_n$) , $\phi \in P[M_m, M_n]$ is decomposable, that is, ϕ is written as the sum of a completely positive map and a completely copositive map, if and only if $\langle A, \phi \rangle \geq 0$ for each $A \in \mathbb{T}$. Note that if a map $\phi \in P[M_m, M_n]$ is completely positive and completely copositive, then $C_\phi \in \mathbb{T}$.

Woronowicz showed that if $m = 2$ and $n \leq 3$, then $M_m^+ \otimes M_n^+ = \mathbb{T}$, and give an explicit example in $\mathbb{T} \setminus M_m^+ \otimes M_n^+$ in the case of $m = 2$ and $n = 4$. This kind of example is called a positive partial transpose entanglement (PPTES) when it is normalised. An example of a PPTES in $(m, n) = (3, 3)$ was firstly given by Choi. A PPTES A is said to be a PPT entangled edge state, or just edge state in short, if there exists no nonzero product vector $x \otimes y \in \mathcal{R}A$ with $\overline{x} \otimes y \in \mathcal{R}A^\tau$, where $\mathcal{R}A$ denotes the range space in A.

In this talk we explain about the classification of PPTES edge states using ranks of A and A^τ. This is a joint work with Seung-Hyeok Kye.

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Skew product graphs, group actions and coactions

David Pask
University of Wollongong, Australia
david.a.pask@gmail.com

A labelling of a directed graph by elements of a group may be used to produce a new graph, called a skew product graph. The resulting graph admits an action this group which is fixed-point free. In this talk we will discuss the applications of this construction for graphs, labelled graphs, higher rank graphs and their C*-algebras.

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Generalized numerical ranges and quantum error correction

Yiu Tung Poon
Iowa State University, Ames, Iowa 50011, U.S.A.
ytpoon@iastate.edu

(Joint work with Chi-Kwong Li)

For a noisy quantum channel, a quantum error correcting code exists if and only if the joint higher rank numerical ranges associated with the error operators of the channel is non-empty. In this talk, we discuss the geometric properties of the joint higher rank numerical ranges and their implications to quantum computing. It is shown that if the dimension of the underlying Hilbert space of the quantum states is sufficiently large, the joint higher rank numerical range of operators is always star-shaped and contains a non-empty convex subset. In case the operators are infinite dimensional, the joint infinite rank numerical range of the operators is a convex set lying in the star center of all joint higher rank numerical ranges, and is closely related to the joint essential numerical ranges of the operators. In addition, equivalent formulations of the join infinite rank numerical range are obtained. As by products, previous results on essential numerical range of operators are extended.

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Complete compactness in abstract harmonic analysis

Volker Runde
University of Alberta, Canada

It has been unknown to this day if $AP(\hat{G})$, the space of all almost periodic functionals on the Fourier algebra $A(G)$ of a locally compact group G is a C^*-subalgebra of the group von Neumann algebra $\mathrm{VN}(G)$, except in a few cases such as if G is almost abelian or both discrete and amenable. In his Diplomarbeit of 1982, under the supervision of G. Wittstock, H. Saar introduced the notion of complete compactness—a variant of compactness that takes operator space structures into account—, which, in turn, enables us to define the notion of completely almost periodic functionals on completely contractive Banach algebras. We will show that, for a Hopf-von Neumann algebra (M, Γ) with M injective, the space of all completely almost periodic functionals on the completely contractive Banach algebra M_* forms a C^*-subalgebra of M.

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Quotients of Fourier algebras and representations that are not completely bounded

Ebrahim Samei
University of Saskatchewan, Canada

In this talk, we observe that for a large class of non-amenable groups G, one can find bounded representations of $A(G)$ on Hilbert space which are not completely bounded. We also consider restriction algebras obtained from $A(G)$, equipped with the natural operator space structure, and ask whether such algebras can be completely isomorphic to operator algebras; partial results are obtained, using a modified notion of Helson set which takes account of operator space structure. In particular, we show that if G is abelian, then the restriction algebra $A(E)$ is completely isomorphic to an operator algebra if and only if E is finite.

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Operator theory and the reproducing kernel of the Dirichlet space

Michio Seto
Shimane University, Matsue 690-8504, Japan
mseto@riko.shimane-u.ac.jp

We study operators satisfying the following inequality:

$$\sum_{n=0}^{\infty} d_n T^n T^{*n} \geq 0, \text{ where } \sum_{n=0}^{\infty} d_n \overline{\lambda}^n z^n = \overline{\lambda} z / \log(\frac{1}{1-\overline{\lambda} z}).$$

In this talk, these operators will be called Dirichlet contractions. Some topics including von Neumann's inequality and Arveson curvature for Dirichlet contractions are discussed.

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Normal Singular Integral Operators with Cauchy Kernel on L^2

Takanori Yamamoto
Department of Mathematics, Hokkai-Gakuen University, Sapporo 062-8605, Japan
yamamoto@elsa.hokkai-s-u.ac.jp

This is a joint work with Takahiko Nakazi. Let P be an orthogonal projection from L^2 onto the Hardy space H^2, and let $Q = I - P$ where I is an identity operator on L^2. For α and β in L^∞, $S_{\alpha,\beta} = \alpha P + \beta Q$ is called the singular integral operator. In this paper, we study the normality of $S_{\alpha,\beta}$.

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On the Aluthge transform of weighted shifts

Jasang Yoon
The University of Texas-Pan American, USA
yoonj@utpa.edu

The concept of subnormality was first introduced by P. Halmos [1950]. The Aluthge transform of a bounded operator on a Hilbert space was first studied in A. Aluthge [1990] and has received much attention in recent years. In this talk we introduce some properties of the k-hyponormality and the subnormality of Aluthge transforms of weighted shifts. The main tools for our results are the Polar Decomposition, the Moore-Penrose inverse and the Smul'jan method for the positivity. This talk is based on a recent joint work with S. H. Lee and W. Y. Lee.