Products of pairs of commuting d-tuples of Banach space operators satisfying an m-Isometric property

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ABSTRACT

A pair (A, B) of Banach operators $A, B \in B(\mathcal{X})$ is m-isometric, $(A, B) \in m$-isometric, if $\triangle^m_{A,B}(I) = (I - L_A R_B)^m(I) = \sum_{j=0}^{m}(-1)^j \begin{pmatrix} m \\ j \end{pmatrix} A^j B^j = 0$; $L_A(X) = AX$ and $R_B(X) = XB$. Extending this definition to commuting d-tuples of Banach space operators, and defining multiplication $\mathbb{AS}$, resp. $\mathbb{A} \bullet \mathbb{S}$, of $\mathbb{A} = (A_1, \cdots, A_d)$ and $\mathbb{S} = (S_1, \cdots, S_d)$ by $\mathbb{AS} = (A_1S_1, \cdots, A_1S_d, \cdots, A_dS_1, \cdots, A_dS_d)$, resp. $\mathbb{A} \bullet \mathbb{S} = (A_1S_1, \cdots, A_dS_d)$, we prove that "if $\mathbb{A}$, $\mathbb{B}$, $\mathbb{S}$, $\mathbb{T}$ are commuting d-tuples satisfying $[\mathbb{A}, \mathbb{S}] = [\mathbb{B}, \mathbb{S}] = [\mathbb{B}, \mathbb{T}] = 0 = \triangle^m_{\mathbb{A},\mathbb{B}}(I) = \triangle^n_{\mathbb{S},\mathbb{T}}(I)$, then $\triangle^{m+n-1}_{\mathbb{AS},\mathbb{BT}}(I) = 0$". Here $[\mathbb{A}, \mathbb{B}] = 0$ means $\mathbb{A}$ and $\mathbb{B}$ commute. Again: "if $\mathbb{A}, \mathbb{B}, \mathbb{S}$ and $\mathbb{T}$ are such that $[\mathbb{A}, \mathbb{S}] = [\mathbb{B}, \mathbb{S}] = [\mathbb{B}, \mathbb{T}] = [\mathbb{S}, \mathbb{T}] = 0 = \triangle^m_{\mathbb{A},\mathbb{B}}(I) = \triangle^{n_i}_{S_i,T_i}(I)$ for all $1 \leq i \leq d$, then $\triangle^{m+\sum_{i=1}^{d} n_i - d}_{\mathbb{A}\bullet\mathbb{S},\mathbb{B}\bullet\mathbb{T}}(I) = 0$."

REFERENCES

[1] O.A.M. Sid Ahmed, M.Cho and J.E. Lee, *On (m, C)-isometric commuting tuples of operators on a Hilbert space*, Res.Math. (2018) 73:51. (doi:10.1007/s00025-018-0810-0)

[2] B.P. Duggal and I.H. Kim, *Structure of elementary operators defining m-left invertible, m-selfadjoint and related classes of operators*, J. Math. Anal. Appl. **495**(2020), 124718.

[3] B.P. Duggal and I.H. Kim, *Isometric, symmetric and isosymmetric commuting d-tuples of Banach space operators*, arXiv 2209.13368.